Communicating expected uncertainty of a geostatistical survey to support co-design with users of information

Christopher Chagumaira[1,2,3,4], Joseph G. Chimungu[3], Patson C. Nalivata[3], Martin R. Broadley[2], Alice E. Milne[2], and R. Murray Lark[1,2]

[1]School of Biosciences, University of Nottingham, Sutton Bonington Campus, Loughborough, LE12 5RD, United Kingdom
[2]Rothamsted Research, Harpenden, AL5 2JQ, United Kingdom
[3]Lilongwe University of Agriculture and Natural Resources, Bunda College, P.O. Box 219, Lilongwe, Malawi
[4]Agri-Food and Biosciences Institute (AFBI), Newforge Lane, Belfast, BT9 5PX, UK

Correspondence: Christopher Chagumaira (christopher.chagumaira@afbini.gov.uk; christopher.chagumaira@nottingham.ac.uk)

Abstract. Much research has examined communication about the uncertainty in spatial information to users of that information, but an equally challenging task is enabling those users to understand *a priori* measures of uncertainty for surveys of different intensity (and cost) at the planning stage. While statisticians can relate sampling density to measures of uncertainty such as prediction error variance, these do not necessarily help information users (e.g., agronomists, soil scientists, policy makers,and health experts) to make rational decisions about how much budget should be assigned to field sampling to produce information of adequate quality. In this exploratory study, we considered four ways to communicate uncertainty associated with predictions made based on data from a geostatistical survey, to determine an appropriate sampling density to meet an information user's expectations. The first method, the offset correlation, is a measure of the consistency of kriging predictions made from data on sample grids with the same spacing but different origins. The second and third methods are based on the conditional prediction distribution: the width of prediction intervals and the probability that, at a location where an intervention is required as indicated by the true value of the sampled variable, the spatial prediction from the survey indicates the contrary. Then the implicit loss function is a method which allows the user to reflect on the valuation of losses from decisions based on uncertain information implicit in selecting some arbitrary sampling density. All these methods depend on the model of spatial dependence for the target variable, but interrogate it in different ways and do not provide the same information. The evaluation of the four communication methods was carried out using a questionnaire that gathered the opinions of participants with experience in survey planning, about the effectiveness of the method and the comprehensibility of the uncertainty measure and its trade-off with the sampling effort. Our results show significant differences in how participants responded to the methods, the conditional probability and implicit loss function approaches were not well understood, and the offset correlation was the most understood. During feedback sessions, the information users highlighted that they were more familiar with the concept of correlation, with a closed interval, in this instance, of [0,1] which is likely to account for the more consistent responses under this method. The offset correlation will likely be more useful to information users, with little or no statistical background, who are unable to express their requirements of information quality based on other measures of uncertainty. However, the results should not

be generalised due to the small sample size–there is need for a more in-depth study with a larger sample size to explore this further.

1 Introduction

1.1 Mapping to support decisions: importance of spatial mapping

Spatial information is needed to support decisions at different spatial scales. Approaches such as geostatistical methods and machine learning algorithms, can predict soil or crop properties at unsampled locations. Geostatistical methods explicitly model spatial dependence as a random process (Webster, 2000), and can therefore support sampling design by inferences from the model (McBratney et al., 1981). Collecting spatial information in surveys, for example on crop or soil properties, incurs costs. Although increasing the sampling density improves the quality of geostatistical predictions by reducing their error variance, there are diminishing returns to increased effort. In principle the optimal survey effort is where the marginal costs of the survey match the marginal improvement in the resulting information if these costs and benefits can be expressed in common terms (Lark et al., 2022).

In a geostatistical model, the value of a variable at an unsampled location has a prediction distribution, conditional on the model and the values and spatial locations of the data. However, the variance of this distribution, the kriging variance, is conditional only on the model and the spatial disposition of the sample points, and so can be calculated from the model for any posited set of observations. McBratney et al. (1981) showed how, given a variogram model, ordinary kriging variances could be computed at the cell centres of square sampling grids of different spacing. This requires information on the variogram before sampling, which is a challenge, but various methods such as expert elicitation (Truong et al., 2013), literature review (Paterson et al., 2018), reconnaissance surveys (Lark et al., 2017), or data from similar areas (Alemu et al., 2022), can be used for estimation. The general approach to sampling design for ordinary kriging, which McBratney et al. (1981) developed, can also be extended to the more general case of spatial prediction from a mixed linear model with spatially correlated random effects and fixed effects, which include covariates such as remote sensor measurements, variables derived from digital terrain models and factorial covariates such as soil maps.

1.2 Communicating the uncertainty of spatial information from proposed survey designs

Despite this effort to address the statistical component of survey planning, the generation of measures of uncertainty for particular proposed designs, little attention has been given to how these measures are understood by information users, such as survey sponsors, who set budgets, and influence data quality. Chagumaira et al. (2021) showed that non-statisticians often find the kriging variance difficult to interpret, and this is consistent with other findings on interpretation of variances by non-specialist (e.g. Konovalova and Pachur, 2021; Weber et al., 2004). It is unlikely that they would find it useful as a measure of the quality of survey output to balance against costs.

This study engaged information users (soil science, agronomy, nutrition, and public health) to evaluate how they interpret measures of survey quality, and their suitability for guiding a decision on the density of samples to be required for a survey. We considered measures derived from an initial variogram of the target variable, and we outline them briefly here, more detail is given in Supplement.

1.3 Proposed methods for communicating information quality

The first measure which we considered is the offset correlation. This is a measure of the consistency of spatial information produced when surveying at a particular grid spacing. Lark and Lapworth (2013) considered a hypothetical case in which a variable is mapped by ordinary kriging from data on a sample grid of spacing ζ, a second map is then made of the same variable and from a grid of the same spacing, but in which the origin is shifted from the original grid by $\zeta/2$ in each direction. They showed how, for a specified variogram, the correlation of the mapped values at some location increases as the sampling density is increased. We suggested that this minimum offset correlation (at a location furthest from a sample point in either grid) is an intuitive measure of the quality of a survey output, it shows the extent to which the mapped value of the variable is robust to the location selected as the origin of the survey grid.

We consider two measures derived from the kriging variance as measures of information quality. The second is the prediction interval, the interval which includes the unsampled value with some specified probability. Prediction intervals for surveys on grids of different spacing were proposed, in visual form which allowed the user to evaluate them relative, for example, to differences between critical values of the target variable for management purposes. The third measure was based on the probability that a the spatial prediction at a location which requires some intervention (because the variable of interest exceeds or falls below a threshold) will indicate the contrary. This was proposed because previous work showed that information users were generally receptive to presentations of uncertain information based on the probability that the mapped variable falls above or below a significant threshold (Chagumaira et al., 2021).

The fourth measure which we considered is based on value of information theory (Journel, 1984; Lark et al., 2022). It is the implicit loss function (Lark and Knights, 2015). A loss function represents the loss incurred when a decision is based on spatial information which is correct (loss $=0$) or in error (loss ≤ 0). This is used to analyse quality of information in cases where losses are reasonably straight forward to specify for different scenarios (e.g. Ramsey et al., 2002). Lark and Knights (2015) proposed that, for more complex cases, the implicit loss function might be used in critical assessment of a specified level of survey effort, based for example, on a fixed budget. An implicit loss function is one which, given a model of survey logistics, and statistical information (such as a variogram when the information is obtained by geostatistical prediction) makes a specified survey density the rational choice, i.e. the choice under which a marginal increase in survey cost is equal to the marginal reduction in expected loss when decisions are based on the resulting information. Lark and Knights (2015) proposed that reflection on the implicit loss function would help information users to decide whether a proposed survey budget is consistent with information users' views on the implications of making decisions with uncertain information, and we evaluated that here.

Decision-making in the presence of uncertainty is complex, and we have examined it on other related settings see Chagumaira et al. (2022). Our focus in this paper is not the decision-making process (sampling density per se), but on the clarity and use-ability of the uncertainty measures for different sampling densities. To do this we presented participants with an explanation of the method. We provided them with the information according to each method on the relationship between sampling density and prediction uncertainty for soil pH and grain Se ($\mathrm{Se_{grain}}$) concentration. The methods each provide a measure of uncertainty of the predictions as a function of sampling density. All the measures depend on a common spatial linear mixed model for the variable, and some also on the location of the marginal distribution. They are therefore mutually consistent, but they do not provide the same information. Our focus was on accessibility of these methods to information users. The actual decision process making would be very case-specific, and we would expect that not all of the methods here are relevant in any one case. We consider this further in the light of our results, in the discussion section.

2 Materials and methods

We engaged with information users from multiple disciplines (e.g. agriculture, soil science, human nutrition, and public health) who were involved with the GeoNutrition project (http://www.geonutrition.com/), which examined strategies to alleviate micronutrient deficiencies (MND) in Ethiopia and Malawi. The project included surveys to provide baseline information on micronutrient concentrations in staple crops and soils, and soil properties (such as pH) which influence soil to plant transfers of micronutrient. Given that concentrations of micronutrients in staple crops and in soils vary spatially, as do biomarkers for micronutrient status and so interventions to address the deficiencies should be based on spatial information (Gashu et al., 2020; Botoman et al., 2022). The spatial information must be interpreted by information users from different disciplines, and all of them might also contribute to decisions on the amount of effort to be expended on field survey. It is plausible that experts with diverse backgrounds prefer different methods for expressing uncertainty, and so we recruited a multidisciplinary panel to explore their perspectives on survey quality and sampling density.

Panel members were drawn from institutions collaborating on the GeoNutrition project, the allied Translating GeoNutrition project in Zimbabwe (ZimGRTA) and the University of Zambia. They were volunteers from sub-national and national-institutions responsible for agricultural research and extension services, public health, and nutrition policy. Soil scientists from the UK were also included. Panel members were invited by email from the local GeoNutrition/ZimGRTA lead, and we had 26 participants (18 were agronomists or soil scientists and 8 public health or nutrition specialists). Most were familiar with the GeoNutrition project and had experience advising on research for policy implementation related to micronutrient supply and crop production in their respective countries. Given our focus on the accessibility of information about sampling intensity and uncertainties,their backgrounds were more relevant than the specific context.

2.1 Format of the exercise

We aimed to gather insights from a diverse group of information users on the effectiveness of proposed methods in evaluating the implications of uncertainty in spatial prediction, specifically as influenced by sampling. This was examined in the context of measuring a soil property and a crop micronutrient. We sought to explore how the information from the sampling conducted in the GeoNutrition project could be used to support decisions about sampling density for other similar projects. We used data from the GeoNutrition project (Gashu et al., 2021), crop and soil properties were measured at national scale in a geostatistical survey conducted in Malawi. We used variograms for soil pH and $\mathrm{Se_{grain}}$ concentration to obtain sampling densities for further notional sampling for an administrative district in Malawi, Rumphi District, with an area of 4769 $\mathrm{km^2}$. The outputs were presented to participants in poster format through PowerPoint, and examples of the posters are shown in Figures S5–S10 in the Supplement.

The elicitation was conducted online using Zoom Video Communications (2022) in two sessions, 26^{th} and 28^{th} April 2022. There were two sessions in order to accommodate participants from different time zones, and to manage the participants in smaller groups to allow for questions and feedback. The invited participants self-identified as (i) agronomist or soil scientist or (ii) public health or nutrition specialists. The participants also self-assessed their statistical/mathematical background and their frequency of use of statistics in their job role (perpetual, regular and occasional use).

In the exercise, an introductory talk was given to explain the study's objectives. During the talk, we explained the four test methods and how they can be used to assess the implications of uncertainty in spatial predictions to determine appropriate sampling grid space for a geostatistical survey. We explained the structure of the questionnaire to the participants. We emphasized to the participants that we were not testing their mathematical/statistical skills and understanding but rather were testing the accessibility of the methods. We had a feedback session to allow the participants to seek clarification on the presented methods.

The participants considered each method in turn and were asked to select an appropriate sampling grid density based on the method. Evaluation of the test methods was done through a questionnaire, as shown on Table 1. Using the first four questions, Q1 to Q4, we wanted to find out if the method helped the participants to identify a sampling grid spacing. On Q5, we wanted the participants to assess the test methods in terms of their effectiveness in finding an appropriate grid spacing. We asked the participants to rank these methods based on their effectiveness to enable a user consider the trade-off between sampling effort and the uncertainty of the spatial predictions in the final survey product. We asked them to put rank 1 as the most effective method and rank 4 the least. The participants recorded their responses using an online questionnaire on Microsoft Forms. The offset correlation was the first method presented to the participants. This was followed by prediction intervals and conditional probabilities. The implicit loss function was the final method presented to the participants. We started with a measure we thought all our stakeholders would most easily understand and then moved on to the more complex methods so as not to undermine their confidence to participate in the exercise.

2.2 Test Methods

We used data collected in the GeoNutrition survey and refer the reader to Gashu et al. (2021) and Botoman et al. (2022) for detailed description of the field sampling. We undertook exploratory analysis of soil pH and $\mathrm{Se_{grain}}$ concentration using QQ plots, histograms and summary statistics to check whether there was need for transformation of the variables for the assumption of normality. The data for $\mathrm{Se_{grain}}$ concentration were skewed and it was necessary to transform them to natural logarithms. The variance parameters for both soil pH and $\mathrm{Se_{grain}}$ concentration were estimated by residual maximum likelihood using the likfit procedure in the geoR packages (Diggle and Ribeiro, 2010) for the R platform (R Core Team, 2022) with a constant mean as the only fixed effect. These variance parameters were used in the subsequent test methods. The thresholds we considered, in

Table 1. The list of questions used to elicit stakeholder opinions about the set of methods that can help end-users to assess the implications of uncertainty in spatial prediction in as far as this is controlled by sampling.

Number	Question	Responses
Q1	We show you here some pairs of example maps of soil pH/Se_{grain}, each pair being based on a different grid spacing, and we also show scatter plots which illustrate the strength of the correlation so with a different offset correlation. What do you think is the smallest correlation that would be acceptable if one of the maps were to be used to make decisions?	(1) 0.4 (2) 0.5 (3) 0.6 (4) 0.6 (5) 0.8 (6) 0.9
Q2	You are shown different scenarios for the prediction of soil pH/Se_{grain} from different grid spacing's, which determine the width of the prediction interval. What is the grid spacing that gives the widest prediction interval that would be acceptable if one of the maps were to be used to make decisions?	(1) Spacing-20km (2) Spacing-40 km (3) Spacing-60 km (4) Spacing- 80 km (5) Spacing-100 km (6) Spacing-120 km
Q3	At some location on the map the true value of soil pH/Se_{grain} indicates that an intervention is required, due to error in prediction there is a non-zero probability that the mapped soil pH/Se_{grain} does not show this, this probability increases with grid spacing as shown on the graph. What grid spacing do you think corresponds to the largest acceptable value of this probability?	(1) Spacing-20 km (2) Spacing-40 km (3) Spacing-60 km (4) Spacing-80 km (5) Spacing-100 km (6) Spacing-120 km
Q4	We have three specified implicit loss functions for predictions Se_{grain} concentration which indicate the expected total loss over an area of 4,769 square kilometres (km^2) for a district/administrative region. The decision for an intervention is based on predictions Se_{grain} concentration with a specified error distribution. Given the likely implications of the error, which of the three functions do you think best represents the magnitude of losses incurred by basing the decision on information with error?	(1) Spacing-10 km (2) Spacing-20 km (3) Spacing-30 km
Q5	Please rank these methods in an order of their effectiveness, in your experience, in terms of finding a level of uncertainty that you are able to tolerate when deciding about a sampling grid density.	Rank 1 being the MOST effective and Rank 4 the least

this study for the prediction intervals and conditional probabilities were soil pH of 5 and $\mathrm{Se}_{\mathrm{grain}}$ concentration of 38 µg kg^{-1}. The threshold for soil pH is 5 in Malawi, such that if the pH at a location falls below 5, it would be necessary to apply lime (Chilimba et al., 2013). The threshold $\mathrm{Se}_{\mathrm{grain}}$ concentration is 38 µg kg^{-1}, such that a serving of 330g of grain flour provides a third of the daily estimated average requirement of Se for an adult woman (Chagumaira et al., 2021). The intervention for soil pH was liming, and $\mathrm{Se}_{\mathrm{grain}}$ was provision of fortified food. Selenium is an essential micronutrient with critical roles in human health, and lack of it can cause thyroid dysfunction, and suppressed immune response (Fairweather-Tait et al., 2011). The methods presented to the participants, used the variance parameters modelled above.

2.2.1 Offset Correlation

We presented the participants with correlated pairs of hypothetical maps of soil pH and $\mathrm{Se}_{\mathrm{grain}}$, with differing correlations, so that the extent to which maps might differ as a result of the grid offset could be visualized. We also showed scatter plots that illustrated the strength of the correlation. Figure 1, shows an example of pair of hypothetical maps of soil pH and the corresponding scatter-plot (see also Figures S5 and S6). The correlation plots showed the kriging predictions for soil pH or $\mathrm{Se}_{\mathrm{grain}}$ concentration predicted with variance parameters estimated in Section 2.2. We asked the participants to identify the smallest offset correlation that would be acceptable if one of the maps were to be used to make decisions based on the soil or grain property (see Table 1). Neither of the posited pair of maps, based on offset grids, is to be regarded as closer to reality than the other, the question is how consistent they are.

2.2.2 Prediction intervals

Using the variance parameters estimated in Section 2.2, we evaluated kriging variances at the centres of cells of square grids of different spacings. We considered minimum and maximum grid spacings of 0.05 and 125 km, respectively, with an increment of 0.5 km. We then computed the cell-centred block kriging variance the spacings we were considering by block kriging (Webster and Oliver, 2007). For all the grid spacings, we computed cell-centred block kriging variance on 0.01 km^2 square blocks. We considered three different predictions for each variable but the prediction interval was fixed, depending only on grid spacing. The three predictions of soil pH were 4.8, 5.5 and 6.0 and those of $\mathrm{Se}_{\mathrm{grain}}$ were 20, 55 and 90 µg kg^{-1}. The predictions of soil pH and $\mathrm{Se}_{\mathrm{grain}}$ concentration were presented to the participants in a chart as shown in Figure 2.

The chart consisted of (a) box plot of the distribution of the measured variable based on all soil samples from the study area, (b) a graph of the lower and upper prediction intervals for the prediction at the point of interest for grid spacings from 0 to 120 km, and lines indicating (c) the z_t and (d) the prediction (see Figures 2, S7 and S8). From the chart, we asked the participants to select the grid spacing that gives the widest prediction interval that would be acceptable if the mapped predictions were to be used to make decisions about soil management or interventions to address human Se deficiency (see Table 1).

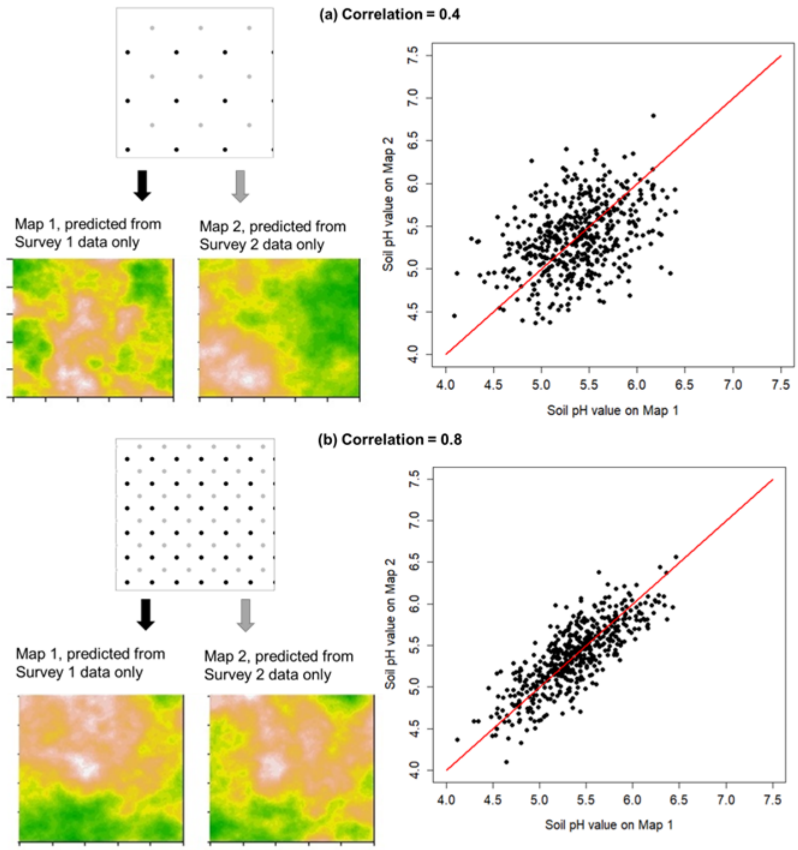

Figure 1. Illustration of the offset correlation (Q1) in two hypothetical cases where this takes values of 0.4 (a) and 0.8 (b). In each case the illustrated subset of grid points is of the same dimensions, so the grid is denser in (b) than (a). In each case a hypothetical data set 1 (black grid points) and set 2 (grey points) is collected from grids of shared spacing but offset north-south and east-west by half the grid spacing. The offset correlation is the correlation of the predictions of the soil properties made at common sites from the two grids. In practice this is calculated theoretically from the variogram and a proposed grid spacing to assess the sensitivity of the original map to variation in the position of the grid origin.

2.2.3 Conditional probability

This is the probability, conditional on the fact that a site requires an intervention (as judged from the true value of the variable being mapped), that the predicted value at the site does not indicate this. It is a useful measure of uncertainty in a case where the overall mean value of the variable is such that the intervention would not be recommended and so the map is important for identifying those locations where it is required. In these conditions the probability of this erroneous conclusion increases as the sample grid becomes coarser and the uncertainty attached to the spatial predictions increases. The probability is bounded on an interval [0,1]. A probability of 1, indicates that the prediction will be equivalent to the overall mean of the dataset. We

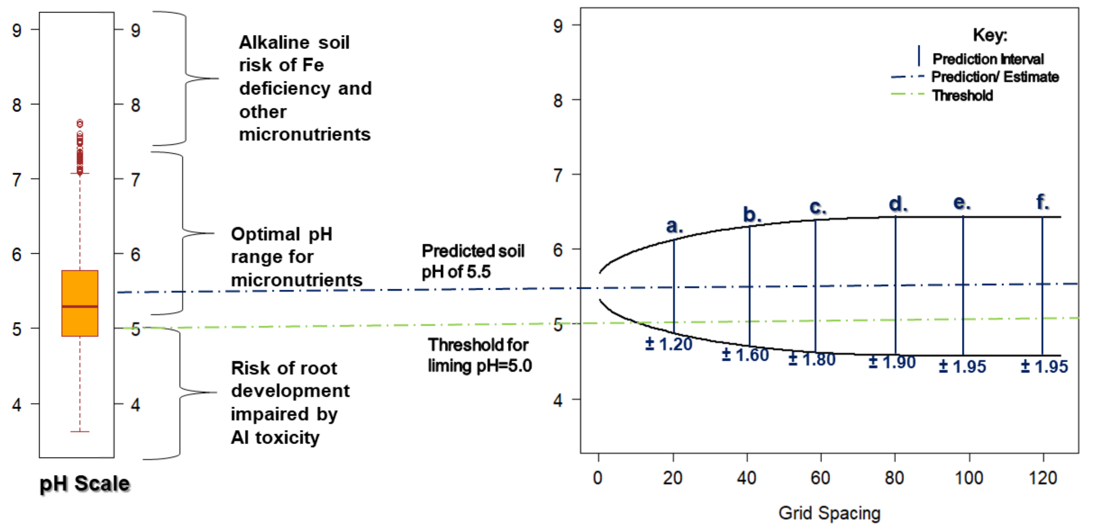

Figure 2. An example of a chart, for prediction intervals (Q2), with prediction of soil pH of 5.5 with prediction intervals and in relation to a threshold of pH = 5.0.

are not aware that this conditional probability has been used before, its derivation from the geostatistical model is provided in the Supplement.

We presented the participants with a graph of the conditional probabilities plotted against grid spacing which is shown in Figure 3 (see also Figure S9). If the prediction of $\mathrm{Se}_{\mathrm{grain}}$ or pH was below the threshold, z_t, an intervention is needed. We then asked the participant what grid spacing they thought corresponded to the largest acceptable value of this probability (see Table 1).

2.2.4 Implicit loss functions

In order to compute the implicit loss function, we needed a cost model for Rumphi district. We used the function defined in Lark and Knights (2015) to return the costs of n samples over an area A km^2, i.e. a sample density of $r = N/A$ samples per km^2 :

$$\mathrm{C}(n) = \omega + vAr + \beta A t_r, \tag{1}$$

where ω are the fixed costs, v cost of laboratory analysis per unit, and β the field costs per work day per team. The quantity t_r is time taken to sample per km^2 at a density of r per km^2. We obtained these costs for Rumphi district by considering the

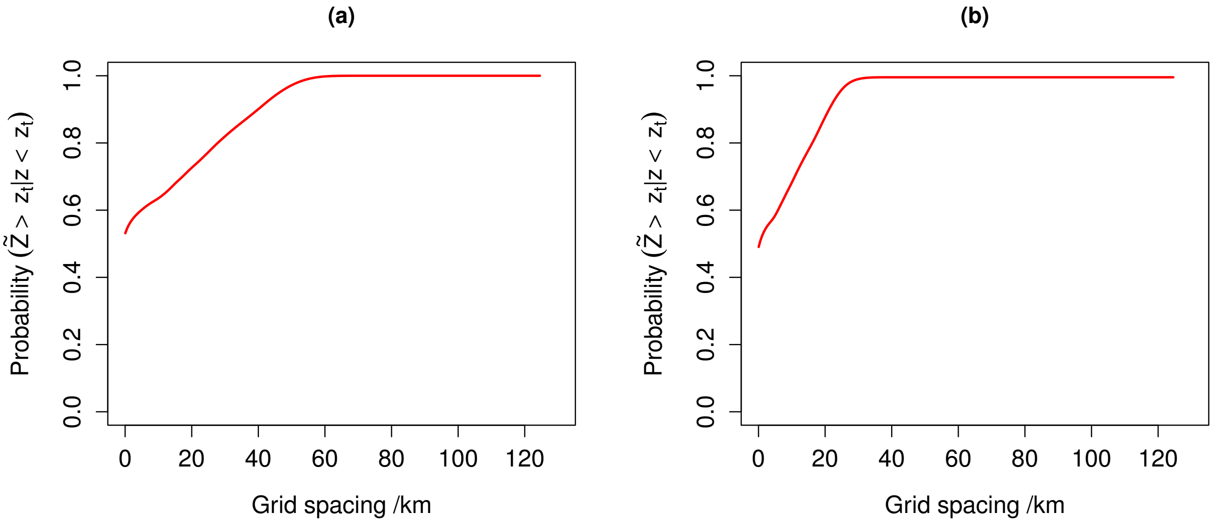

Figure 3. An example of the chart of conditional probabilities (Q3) plotted against grid spacing for (a) soil pH and (b) $\mathrm{Se_{grain}}$ concentration. At a location $\mathbf{x}_0$, $\tilde{Z}$ is the prediction and z is the value of the variable at that location.

available costs for crop sampling during the GeoNutrition survey conducted in Malawi at national-scale (Gashu et al., 2021; Kumssa et al., 2022). A detailed description of how the costs were computed is presented in the Supplement.

We fixed the asymmetry ratio at 1.5, assuming the elicited mean probability threshold from similar stakeholders in Ethiopia and Malawi (Chagumaira et al., 2022) can be regarded as

an approximation of P_0 which corresponds to a quantile of prediction distribution. This implied a bigger loss for overestimation of the variables (i.e. failing to intervene of $\mathrm{Se_{grain}}$ are smaller than prediction) than for underestimation. With the implicit loss function we assumed that the sample density is fixed (e.g. on budgetary grounds) and computed the loss function which would make that a rational choice. We presented three specified implicit loss functions for predictions of $\mathrm{Se_{grain}}$ for Rumphi district, with an area of 4,769 km^2 with sampling densities fixed at 10, 20 and 40 km. Figure 4 (see also Figure S10), shows the implicit loss function for $\mathrm{Se_{grain}}$. We then asked the participants to identify the loss function implied by the sampling decision that looked more plausible to make decisions about interventions to address human Se deficiency (see Table 1).

2.3 Data Analysis

2.3.1 Test methods

The responses for Q1 to Q4 were presented as contingency tables. The contingency table allowed us to evaluate whether there were differences in the responses of the participants based on (i) variable used in the test method, (ii) professional group and (iii) by frequency of use of statistics. We analysed the contingency table on the basis of a null hypothesis that the distribution of

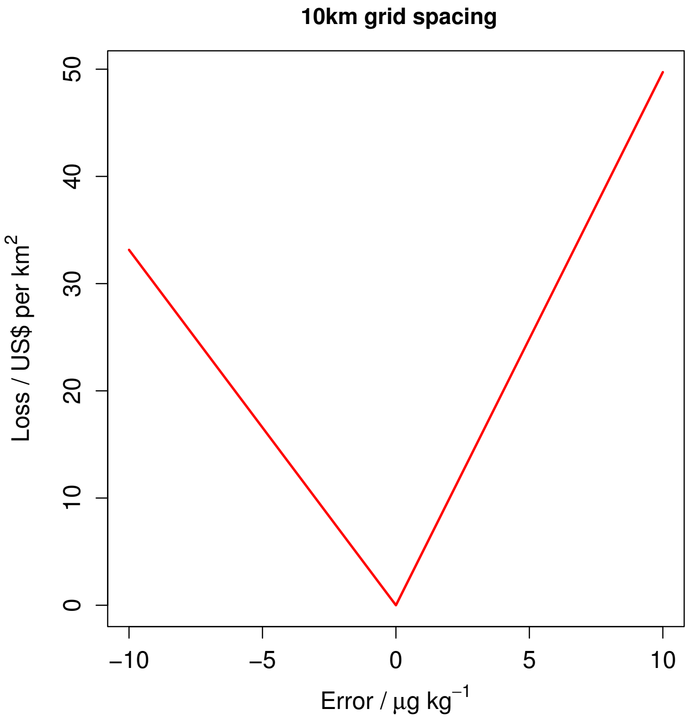

Figure 4. An example of specified implicit loss functions (Q4) for predictions of $\mathrm{Se}_{\mathrm{grain}}$ concentration at a 10 km grid spacing.

observations between responses (e.g. selected grid spacing) is independent of the factor in the column (e.g. professional group). If evidence is provided to reject the null hypothesis, then this would indicate that how a respondent interprets the information presented to select a grid spacing depends on their professional group. A detailed description of how we used the contingency tables to partition the responses of the questionnaire are presented in the Supplement.

The rows of each table correspond to the response (e.g. the different grid spacings) and, the full table, the columns correspond to the frequency of use of statistics, nested, within professional group and nested within variable used (soil pH or $\mathrm{Se}_{\mathrm{grain}}$). Contingency tables allowed us to test the null hypothesis of random association of responses with the different factors in the columns. The expected number of responses under the null hypothesis, $e_{i,j}$ in a cell $[i,j]$, is a product of row (n_i) and column (n_j) totals dived by the total number of responses (N), and this the null hypothesis of the contingency table which is equivalent to an additive log-linear model of the table. An alternative to the additive model for the contingency table, is the saturated model that has an extra $(n_r - 1)(n_c - 1)$ term that allows for interaction amongst the columns and tables of the table. The proportions of observed responses $o_{i,j}$ may differ from $e_{i,j}$ in a cell $[i,j]$ and the likelihood ratio statistic or deviance, L, can be used to provide evidence against the null hypothesis. The likelihood ratio statistic is computed by

$$L = 2\sum_{i=1}\sum_{j=1} o_{i,j} \log \frac{o_{i,j}}{e_{i,j}}. \qquad (2)$$

where L has an approximate χ^2 distribution under the null hypothesis of random association between the rows and columns of the table, with $(n_r - 1)(n_c - 1)$ degrees of freedom (Christensen, 1996; Lawal, 2014). We fitted the log-linear models using the loglm function from the MASS package (Venables and Ripley, 2002) for the R platform.

In our study, we wanted to find out if the responses recorded by the participants depended on the variable used (soil pH or $\mathrm{Se_{grain}}$ concentration), and background of the respondent. We expected the responses to differ. We thought the participants would have different perceptions of the impacts of the uncertainty for soil pH and $\mathrm{Se_{grain}}$ concentration. There were more agronomist or soil scientists than public health or nutrition specialists in the meeting, and we expected the priorities of the groups to differ when making interventions for soil pH and $\mathrm{Se_{grain}}$ concentration. We also thought the frequency of use of statistics would influence the choice of method used to select an appropriate grid spacing.

We first tested for differences in responses recorded for each test method, by the variable used (soil pH or $\mathrm{Se_{grain}}$ concentration) using contingency tables. The responses from stakeholders in different professional groups were pooled within the two variables, as illustrated by pooled table 1 on Table S5 in the Supplement. This gave us a six (responses) by two (variables) contingency table with 5 degrees of freedom for the questions corresponding to offset correlation, prediction intervals and conditional probabilities (Q1 to Q3). However, for the implicit loss function we did not consider this because we only had a loss function for $\mathrm{Se_{grain}}$ concentration.

Second, we considered if the differences in the responses depended on the professional group of the respondent. Finally, we considered whether the frequency of use of statistics in their job role had an impact on the responses recorded by the respondents. For some questions, we noted difference in the responses when pooled within variable used (soil pH or $\mathrm{Se_{grain}}$ concentration) and there was no differences in responses in professional groups and frequency of use of statistics for all questions. We further analysed the pooled tables or separate subtables to examine if the responses were uniformly distributed and the null hypothesis is a random distribution. We wanted to test whether the responses of the participants were uniform, i.e., each grid spacing has equal likelihood of occurrence.

2.3.2 Assessment of the method

The responses for the Q5 were tabulated with the methods as the columns and ranks as the rows. The participants ranked their preferred method first. However, to calculate the mean rank, $\bar{r}_i$, for each method for all the respondents, we assigned a score of 4 for the most preferred method and 1 for the least. We computed the $\bar{r}_i$ for each method for all respondents. We then separated the respondents by their professional group and computed the mean ranks.

Finally we separated the respondents by their frequency of use of statistics in their job role. Under a null hypothesis of random ranking for set of k ranks, the expected mean rank is $(k+1)/2$. The evidence against this hypothesis is measured a statistic distributed as $\chi^2(k-1)$:

$$\frac{12n}{k(k+1)} \sum_{i=1}^{k} \left\{ \bar{r}_i - \frac{k+1}{2} \right\}, \tag{3}$$

where n is the total number of rankings (Marden, 1996).

Table 2. Analysis of the question on offset correlation, Q1, according to variable used, professional group and frequency of use of statistics.

	Deviance (L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	54.57	55	0.491
Pooled by variable used (pH v. $\mathrm{Se}_{\mathrm{grain}}$)	3.29	5	0.656
Pooled by professional group	6.50	5	0.260
Pooled by frequency of use of statistics	8.35	10	0.595
Subtable–pooled counts: variable used			
Soil pH	27.01	25	0.352
$\mathrm{Se}_{\mathrm{grain}}$	24.2	25	0.507
Subtable–pooled counts: professional group			
Agronomist or soil scientist	26.25	25	0.394
Public health or nutrition specialist	21.81	25	0.646
Subtable–pooled counts: frequency of use of statistics			
Perpetual use of statistics	8.99	15	0.878
Occasional use of statistics	18.17	15	0.254
Regular use of statistics	19.06	15	0.211
Subtable–pooled counts			
Responses are uniformly distributed	17.69	5	0.003

3 Results

There was reasonably even spread in terms of the location of our participants, see Figure S11 (Supplement). About 54% of the participants were constantly using statistics/mathematics within their job role. Only a few participants were educated to the level of certificate/diploma (8%).

3.1 Test methods

3.1.1 Offset correlation

Results for the question (Q1) on offset correlation are shown in Table 2, with the full contingency table in the Supplement (Table S6). Pooled responses showed strong evidence against a uniform distribution ($p = 0.003$). Figure 5 shows how participants responded to Q1. Most selected 0.7 as the minimum acceptable offset correlation if one of the maps were to be used to make decisions based on the soil or grain property. Grid spacings corresponding to this value–25 km (soil pH) and 12.5 km ($\mathrm{Se}_{\mathrm{grain}}$)–were extracted from plots of offset correlation vs. grid spacing (see Figure S4), based on each variable's variogram.

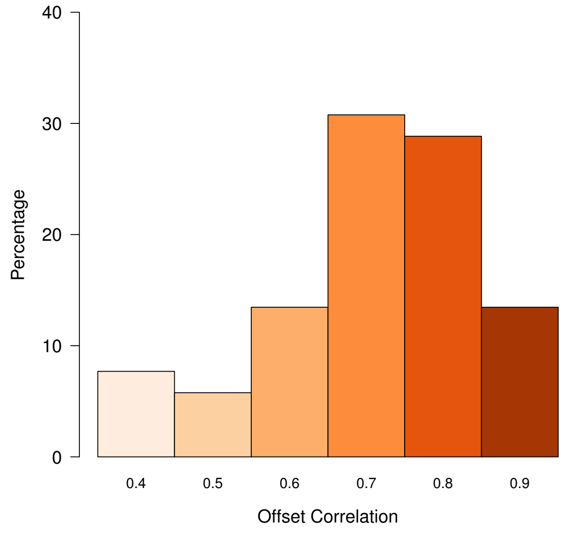

Figure 5. Bar charts showing how the participants responded to Q1 for offset correlation.

3.1.2 Prediction interval

Further analysis of the question (Q2) on prediction interval we used pooled response counts, as responses did not differ by professional group or frequency of statistical use (Table 3). Figure 6 shows how all participants responded to Q2. For this method, there was no clear preference for grid spacing when sampling soil pH and $\mathrm{Se}_{\mathrm{grain}}$, because there was no evidence against a uniform distribution of responses ($p = 0.169$).

3.1.3 Conditional probabilities

There were differences in the responses when the columns were pooled by variable ($p \leq 0.001$, Table 4), so further analysis was done separately for soil pH and $\mathrm{Se}_{\mathrm{grain}}$. For both variables, there was strong evidence to reject the null hypothesis that the responses were uniformly distributed ($p \leq 0.001$). Figure 7a shows responses for soil pH (grid spacing: 60 km), and Figure 7b shows responses for $\mathrm{Se}_{\mathrm{grain}}$ (grid spacing: 40 km).

3.1.4 Implicit loss functions

Further analysis of the question (Q4) on the implicit loss function was based on pooled counts of responses (Table 5). There was strong evidence to reject the null hypothesis that the responses were uniformly distributed ($p \leq 0.001$). Figure 8 shows the participants responses to Q4. The grid spacing chosen by the participants for $\mathrm{Se}_{\mathrm{grain}}$ concentration is 20 km.

Table 3. Analysis of the question on prediction interval, Q2, according to variable used, professional group and frequency of use of statistics.

	Deviance (L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	56.0	55	0.437
Pooled by variable used (pH v. $\mathrm{Se_{grain}}$)	0.972	5	0.965
Pooled by professional group	4.36	5	0.498
Pooled by frequency of use of statistics	14.5	10	0.152
Subtable–pooled counts: variable used			
Soil pH	23.8	25	0.531
$\mathrm{Se_{grain}}$	31.2	25	0.181
Subtable–pooled counts: professional group			
Agronomist or soil scientist	26.5	25	0.381
Public health or nutrition specialist	25.1	25	0.455
Subtable- pooled counts: frequency of use of statistics			
Perpetual use of statistics	9.68	15	0.840
Occasional use of statistics	16.88	15	0.330
Regular use of statistics	15.08	15	0.450
Subtable- pooled counts			
Responses are uniformly distributed	7.77	5	0.169

3.2 Assessment of the test methods

We analysed the responses for Q5 by computing the mean ranks across all participants, by professional group, and by frequency of use of statistics, and tested for evidence against the null hypothesis of random ranking. In all cases, there was strong evidence to reject the null hypothesis of random ranking, $p \leq 0.001$ (Table 6). Overall, offset correlation was consistently ranked as the most effective, while implict loss function was generally ranked as the least effective.

4 Discussion

In this study, we presented to groups of information users, four methods (offset correlation, prediction intervals, conditional probabilities and implicit loss functions) that can be used to support decisions on sampling grid spacing for a survey of soil pH and $\mathrm{Se_{grain}}$. We wanted to find out if the information users had a preference among the approaches presented to them. Offset correlation was ranked first as the method the stakeholders found easy to interpret (see Figure 9) in order to a decision on sampling density. Most respondents (30 %) selected an offset correlation of 0.7, and slightly fewer selected 0.8 so over half

Table 4. Analysis of the question on conditional probabilities, Q3, according to variable used, professional group and frequency of use of statistics.

	Deviance (L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	60.6	55	0.281
Pooled by variable used (pH v. $\mathrm{Se_{grain}}$)	26.7	5	< 0.001
Pooled by professional group	5.32	5	0.378
Pooled by frequency of use of statistics	14.5	10	0.152
Subtable–pooled counts: variable used			
Soil pH	12.1	25	0.986
$\mathrm{Se_{grain}}$	21.8	25	0.647
Soil pH subtable–pooled counts: professional group			
Pooled within professional group	4.48	5	0.483
Agronomist or soil scientist	3.10	10	0.979
Public health or nutrition specialist	4.50	10	0.922
Soil pH subtable–pooled counts: frequency of use of statistics			
Pooled within frequency of use of statistics	0.889	10	1.00
Perpetual use of statistics	4.50	5	0.480
Occasional use of statistics	4.36	5	0.499
Regular use of statistics	2.33	5	0.802
Soil pH subtable–pooled counts			
Responses are uniformly distributed	50.15	5	< 0.001
$\mathrm{Se_{grain}}$ subtable–pooled counts: professional group			
Pooled within professional group	4.77	5	0.445
Agronomist or soil scientist	11.0	10	0.361
Public health or nutrition specialist	6.09	10	0.808
$\mathrm{Se_{grain}}$ subtable–pooled counts: frequency of use of statistics			
Pooled within frequency of use of statistics	9.55	10	0.481
Perpetual use of statistics	1.73	5	0.886
Occasional use of statistics	5.55	5	0.353
Regular use of statistics	4.99	5	0.417
$\mathrm{Se_{grain}}$ subtable–pooled counts			
Responses are uniformly distributed	36.77	5	< 0.001

Table 5. Analysis of the question on implicit loss function, Q4, according to variable used, professional group and frequency of use of statistics.

	Deviance (L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	8.91	10	0.541
Pooled by professional group	0.49	2	0.781
Pooled by frequency of use of statistics	1.49	4	0.828
Subtable–pooled counts: professional group			
Agronomist or soil scientist	2.33	4	0.676
Public health or nutrition specialist	6.09	4	0.193
Subtable- pooled counts: frequency of use of statistics			
Perpetual use of statistics	1.73	2	0.422
Occasional use of statistics	1.73	2	0.422
Regular use of statistics	3.96	2	0.138
Subtable- pooled counts			
Responses are uniformly distributed	54.00	2	< 0.001

Table 6. Analysis of Q5 according to professional group and level of use of statistics in job role

	Test Statistic (X^2)	Degrees of freedom	P^*
All respondents	61.1	3	< 0.001
Professional group			
Agronomist or soil scientist	49	3	< 0.001
Public health or nutrition specialist	15.6	3	< 0.001
Frequency of use of statistics			
Perpetual user of statistics	34	3	< 0.001
Occasional user of statistics	28.5	3	< 0.001
Regular user of statistics	49.8	3	< 0.001

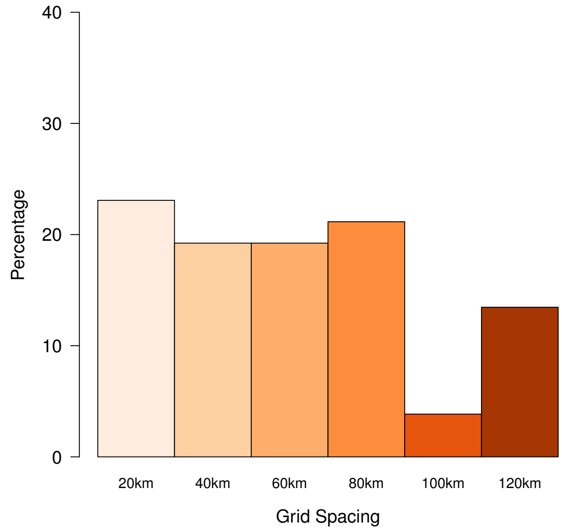

Figure 6. Bar charts showing how the participants responded to the Q2 for prediction intervals.

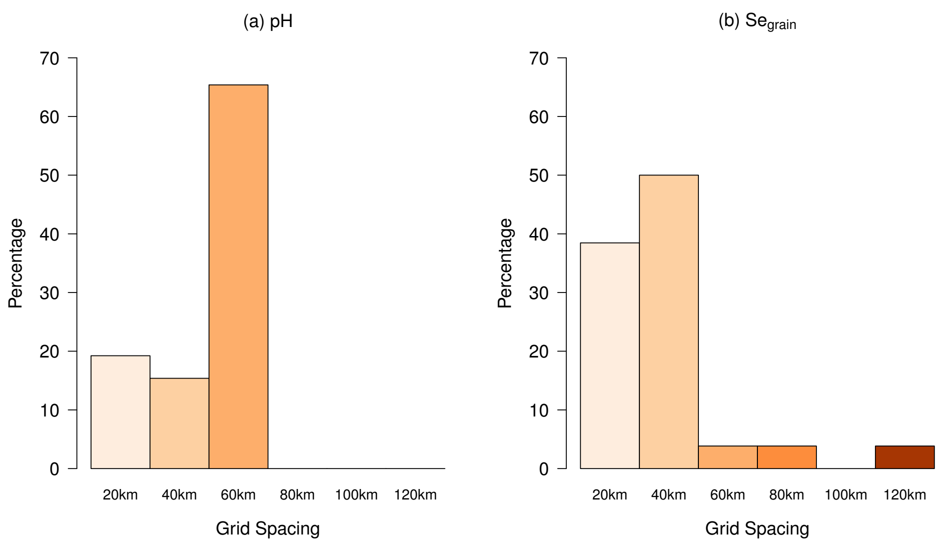

Figure 7. Bar charts showing how all the participants responded to the Q3 for conditional probabilities for (a) soil pH and (b) $\mathrm{Se}_{\mathrm{grain}}$ concentration.

of respondents are accommodated within this range of values. During the feedback session, information users highlighted that they were more familiar with the concept of correlation, particularly within a closed interval of [0,1]. This familiarity may have

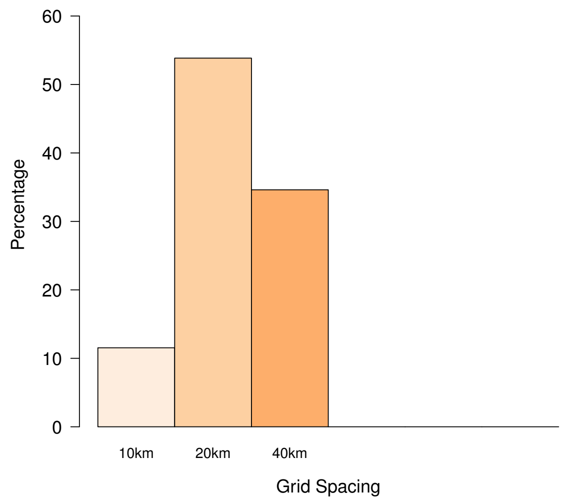

Figure 8. Bar charts showing how all the participants responded to the Q4 for implicit loss function.

influenced the consistency of the results for this criterion, with over half the respondents selection 0.7 or 0.8 as a minimum acceptable correlation. This pattern aligns with Hsee (1998), who found that relative measures of uncertain quantities (e.g. size of a food serving relative to its container) are more readily evaluated than absolute measures (the size of serving). Bounded attributes, such as correlation on a [0,1] scale, can play a more prominent role in individual's judgement of utility. Hsee (1998) describe this as the "relation-to-reference" attribute. This may explain why the offset correlation was highly-ranked.

The offset correlation seems to be a criterion which respondents are more likely to find comprehensible, and so a basis for selecting the sample density for a geostatistical survey, than alternatives such as kriging variance. Furthermore, it appears to be a measure of uncertainty which participants in the study found comprehensible, and so were able to use to select a grid sample spacing. It recognises that spatial variation means that maps interpolated from offset grids will differ but that the more robust the sampling strategy the more consistent they will be. However, Chagumaira et al. (2021) found that measures of uncertainty related to a specific management threshold of the mapped variable were preferred by participants for the interpretation of uncertain spatial information to general quality measures without a specific management or policy implication. In this case, in contrast, the preferred criterion, the offset correlation, is a general measure of map quality, which is not directly linked to specific interpretation.

Conditional probabilities were ranked second. Under this method, the information users selected spacings where conditional probabilities was 1.0 or very close, i.e. the prediction equivalent to the overall mean. This suggests that the information users may not have fully understood the method. This finding is consistent with the general view that users of information commonly find probabilities difficult to interpret (Spiegelhalter et al., 2011). Because probabilities are bounded [0,1], the 'relation-to-

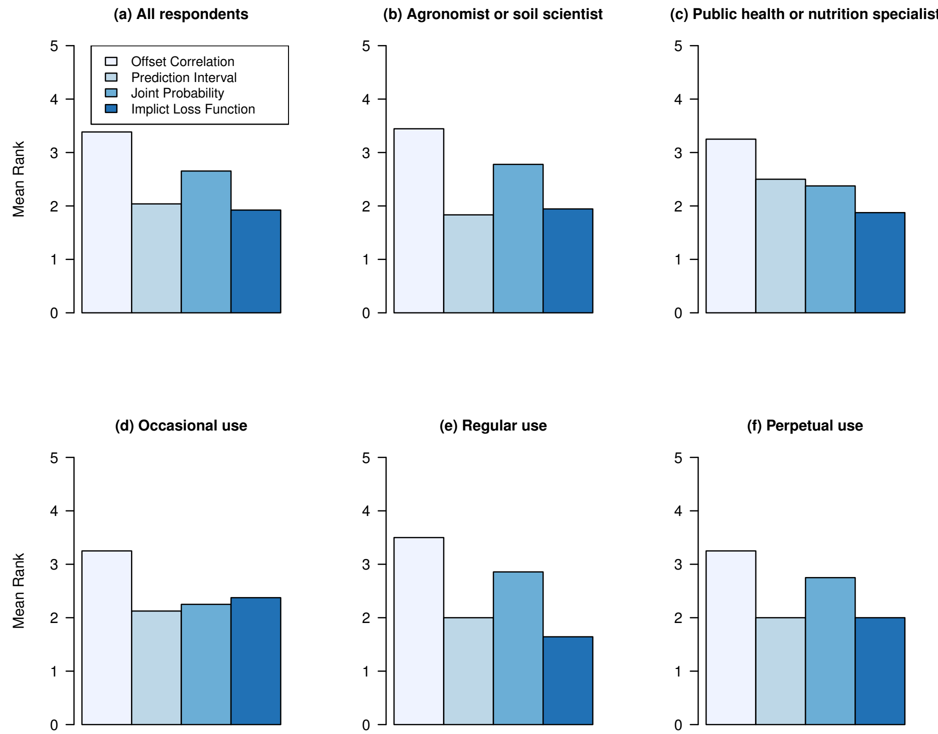

Figure 9. Ranking of test methods in terms on the most effective: (a) by all respondents, professional group: (b) agronomists or soil scientist and (c) public health or nutritionist specialists, and frequency of use of statistics: (d) occasional use, (e) regular use and (f) perpetual use.

reference" attribute effect by Hsee (1998) may explain the previous preference for conditional probabilities (Jenkins et al., 2019; Chagumaira et al., 2021), but information users still struggle to interpret them correctly. More work is needed to investigate whether explaining or framing the conditional probabilities in a different way would improve the judgement of their utility by the information users. For example, if the probability was expressed as "probability of an error of omission at a site where intervention is required. More examples and more illustration may be needed in order to 'prime' the participants before the exercise. A method might be regarded as easy to interpret, because of its form, even when it is not (in this case a large value of the probability indicated that there was no spatial information in the map to make its predictions better than the overall mean).

Prediction intervals were ranked third by all the respondents, but there was no evidence against the null hypothesis of random selection among the available spacings. During a feedback session, the information users cited difficulties of assessing the significance of a given prediction interval given that it can be associated with different prediction values. For very large or small prediction values the uncertainty is immaterial, it is near decision threshold that it becomes important. Similarly, prediction intervals were not highly ranked by information users for communicating uncertainty in maps (Chagumaira et al., 2021). Similar reasons were given by the respondents. We expected that prediction intervals would be of greatest value for specific interpretation of particular sites but would be of limited value for survey planning.

The implicit loss functions was the lowest-ranked method. The group also commented that they had difficulties understanding this method, and most people opted for the central value. Loss functions are not readily accessible. It is difficult to define a loss function because it requires the cost of the errors, and we tried to show information users some consistent approach with some plausible design. The fact that they did not understand the loss functions, shows there is need for more specific examples to help information users think about loss function and their implications. It might help the information users to provide some quantitative information about the costs of the survey, cost associated with intervention campaigns and costs of the impacts on MND on a country's gross domestic production. A reflection of these would allow the information user to use these implicit assumptions when they were making decisions for selecting a fixed grid spacing for working with (Lark and Knights, 2015).

The background of the information users, i.e., professional group and frequency of use of statistics, had no influence on their responses for all the methods. However, the background of the information users had an influence on their ranking of the methods in terms of their effectiveness. The offset correlation was ranked as the most effective by all professional groups and by all respondents separated by frequency of use of statistics. Prediction intervals were ranked least effective by those respondents who identified as agronomist or soil scientist, but were ranked second by those in public health or nutrition.

At the beginning of the online workshop, we explained each method with the aid of illustrations. After an explanation of each method, there was a feedback session to allow the participants opportunities to seek clarity on ambiguous and unfamiliar concepts from the presenters. The participants' questions were answered and explained in different ways by CC, RML and AEM, with the use of illustrations. However, there are limitations with online workshops. Most participants would have the cameras switched off, and the "unconscious" feedback to presenters by observing the reactions of participants could not be noticed as during in-person workshops. The "unconscious" feedback would prompt the presenter to use a different approach to explain unfamiliar concepts and ambiguous terms. Due to internet connectivity, online workshops are timed and there will less time for feedback sessions. In such instances, respondents may seek clarity from the colleagues who have the same interests, resulting in bias (Ball, 2019).

All the methods may give different results for different variables, because they depend on the variogram of the variable in question. There maybe different grid spacings selected for the different variables. A potential problem may exist, if the variables were to be sampled in one survey and what spacing should be used? This is an important question that needs to be addressed when planning for soil and crop sampling. It may be reasonable to opt for the grid spacing for the variable that maybe the hardest to characterise. Another option would to consider some minimum quantile over all variables through a group elicitation. Black et al. (2008) proposed that a critical subset of soil properties are identified such that the overall sampling scheme is satisfactory for all of the so-called 'canary indicators'.

All the information users recruited in this study were employed in public sector institutes (e.g., universities, civil organisations, research, and extension) and had experience in their respective fields in an SSA setting. We had no basis for a power analysis to identify a sample size for this activity. Given the exploratory nature of this research, our primary aim was to capture insights from as many relevant participants as possible within each institution. As a result, our major consideration was

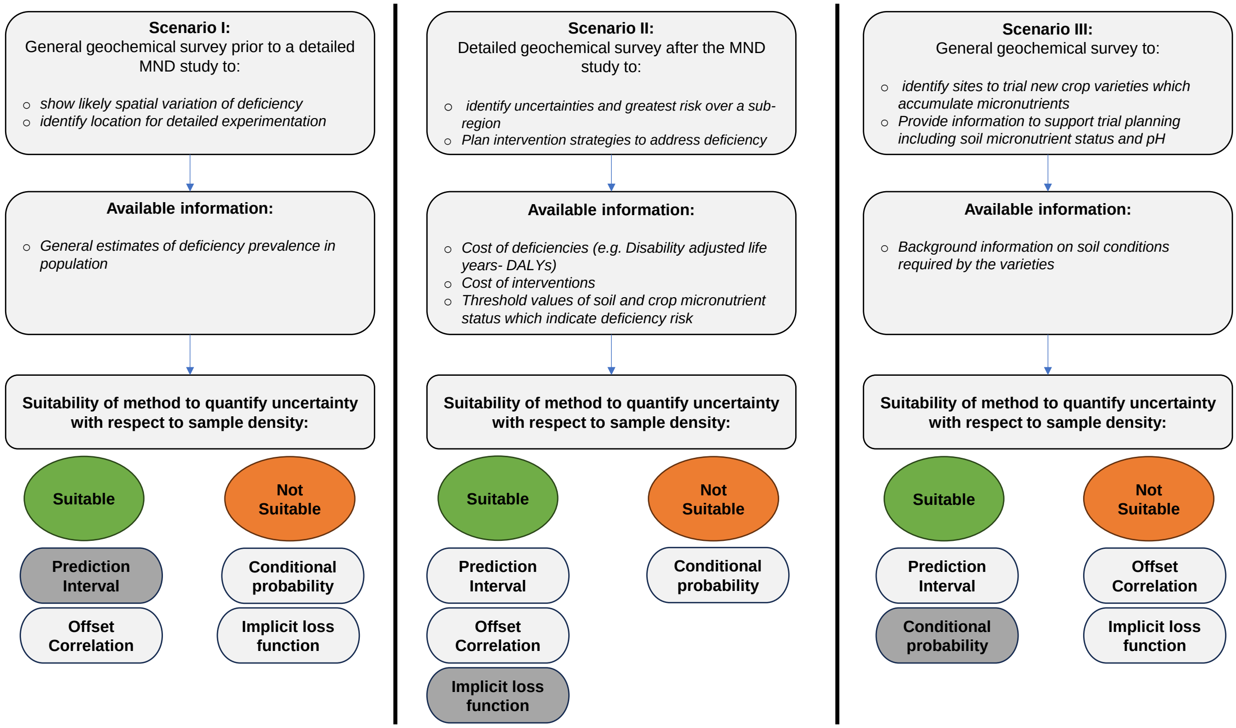

Figure 10. Illustration of the different scenarios of geochemical surveys and suitability of the method to quantify uncertainty with respect to sampling densities.

recruiting individuals willing to participate and with experience in their respective institutions. We therefore attempted to recruit the entire set of suitable respondents in each country. We recognize that the small sample size limits the generalizability of statistical findings. While this study provides insights into participant perspectives by specialism, the lack of demographic information—such as gender, age, location, and years of experience—limits the depth of analysis. These characteristics may impact responses; for example, different age groups or experience levels might prioritize certain issues differently. Future studies should consider including these demographic details to explore how such factors influence perspectives, thus enhancing the robustness of the findings and allowing for subgroup analysis. For this reason, we have interpreted results cautiously and have also incorporated qualitative insights from participants to provide a richer context for understanding these early findings. Moving forward, we plan to include an initial power analysis and possibly extend the study through broader collaborations to enhance robustness.

As we have emphasized, in our objectives, this study was to examine the accessibility of different measures of uncertainty to professional stakeholders. While all methods depend on a common linear mixed model for all the variables, they provide differing, although consistent, information. The different measures would each be most useful for sampling planning for geostatistical prediction to address particular problems. We illustrate this in Fig 10. In scenario I, is a general geochemical survey prior to a detailed MND study to show likely spatial variation of deficiency and identify location for detailed experimentation.

In this case there are no specific decisions to be made using the map and there are no costs (e.g. costs of interventions) linked to the outcomes under considerations. So the implicit loss functions and conditional probabilities (there are no thresholds) will not be useful. However, the prediction intervals and offset correlation will be very useful. Due to the lack of specificity in use of the maps and associated costs the decision will be relatively straightforward for the range of stakeholders in consideration. Scenario II is a detailed geochemical survey after the exploratory MND study with specific objectives to identify uncertainties and greatest risk over a sub-region, and plan intervention strategies to address deficiencies. Here there are quantifiable losses under the errors and the information will be required information important policy-implemented tasks. Given these considerations, the offset correlation will be too general and therefore not very useful. The threshold for making decision to intervene is known and therefore the conditional probabilities will be useable and useful. However, there will be need to invest in specific training for specialists (e.g. public health and nutrition experts), and to elicit a probability threshold that represents a consensus model of the losses associated with making a decision with uncertain data. This could follow Chagumaira et al. (2022), who elicited a mean probability threshold for making decisions for making decisions about interventions to address Se deficiency. The implicit loss function will also be useful in this scenario and can be used to judge resource allocation between this and general food fortification. Scenario III is general geochemical survey to identify sites to trial new crop varieties that accumulate micronutrients, and to provide information to support trial planning such as soil micronutrient status and pH. Background information on soil conditions required by the varieties will be available. In this case, the offset correlation is too general a measure to be useful, and implicit loss functions are not relevant. However, the conditional probabilities will be very relevant, for example, once could compute the probability that a valuable potential site is not identified. The prediction interval will also be useful in this case, but not as direct as the conditional probabilities. There will be need for training give the challenges our participants faced in understanding the method– e.g, use of games and visual examples of everyday problems to build confidence in the interpretation of probabilities to support decision making.

The events were planned prior to the lifting of all COVID-related restrictions on overseas travel from the UK and on larger gatherings in partner countries. Consequently, participant numbers were limited, and we recognize that these results should not be generalised due to the small sample size. To deepen our understanding, especially regarding the impact of professional grouping, a larger-scale elicitation is recommended. Conducting a face-to-face study would also be valuable to ensure participants fully grasp the probability concepts—particularly conditional probability—through interactive activities such as games and quizzes before formal evaluation. A practical takeaway is that more time is needed for participants to become familiar with the methods to improve the quality of the elicitation.

5 Conclusions

In this exploratory study we evaluated four methods of communicating uncertainty associated with kriging predictions made from data from a geostatistical survey, to determine an appropriate sampling density to meet information users expectations. Users of information on soil variation need accessible ways of understanding the implications of sampling designs on spatial prediction and their uncertainties. The background (professional group and frequency of use of statistics) of the information user had no influence in the responses selected for each approach. Of these methods we tested, the offset correlation was most favoured, but had no direct link to decision making and some methods of communication were not well understood (conditional probabilities and implicit loss functions). There were consistent responses under the offset correlation–compared to the other methods, and will likely be more useful to information users, with little or no statistical background, who are unable to express their requirements of information quality based on other measures of uncertainty. Although previous work has found that uncertainty of spatial information is best understood when presented in terms of a decision-specific metric, that was not the case here. This shows that more work must be done to develop and elucidate decision specific approaches, perhaps through methods to elicit useful loss functions. Given the small sample size in this study, there is need for a more in-depth study with a larger sample size to explore these findings further.

Author contributions. The study design was conceived and implemented by CC, RML and AEM. PCN and MRB were responsible for project administration and funding. PCN and JGC supervised the data collection. All authors contributed to the preparation of the article.

Competing interests. The authors declare that they have no conflict of interest.

Disclaimer. The funders were not involved in the design of this study or the collection, management, analysis and interpretation of the data, the writing of the report or the decision to submit the report for publication.

Ethics statement. Ethical approval to conduct this study was granted by the University of Nottingham, School of Biosciences Research Ethics Committees (SBREC202122022FEO) and participants gave informed consent to their participation and subsequent use of their responses. No remuneration was offered, but all participants in African countries who were not able to participate from institutional offices were provided with a one-day data bundle to allow them to join online.

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