Planning a geostatistical survey to map soil and crop properties: eliciting sampling densities

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Abstract. ~~The communication of uncertainty is not only a challenge when an information product has been produced, but also in~~ Much research has examined communication about uncertainty in spatial information to users of that information, but an equally challenging task is enabling those users to understand measures of uncertainty for surveys of different intensity (and so cost) at the planning stage. ~~When planning a survey of soil properties it is necessary to make decisions about the sampling density. Sampling density determines both the quality of predictions and the cost of fieldwork~~While statisticians can relate sampling density to measures of uncertainty such as prediction error variance, these do not necessarily help information users (e.g., agronomists, soil scientists, policy makers and health experts) to make rational decisions about how much budget should be assigned to field sampling to produce information of adequate quality. In this study, we considered four ways ~~in which the relationship between sample density and the uncertainty of predictions can be related based on prior information about the variability of the target quantity. These are offset correlation, prediction intervals, conditional probabilities of the interpretation errors and implicit loss functions. Offset~~ to communicate uncertainty associated with predictions made based on data from a geostatistical survey, to determine an appropriate sampling density to meet an information user's expectations. These include two methods based on the conditional prediction distribution: the width of prediction intervals, and the joint probability that a particular intervention is required at a random location, but the spatial information indicates the contrary. A third method, the offset correlation is a measure of the consistency of kriging predictions made from data on sample grids with the same spacing but different origins. ~~Prediction intervals and conditional probabilities are based on the prediction distribution of the variable of interest. All four of these methods were based on information on soil pH and Se concentration in grain (Se$_{grain}$) in Malawi were presented to a group of stakeholders, who were asked to use them in turn to select a~~ The implicit loss function is a method which allows the user to reflect on the valuation of losses from decisions based on uncertain information implicit in selecting some arbitrary sampling density. ~~Their responses were evaluated and they were then asked to rank the methods based on their effectiveness, in their experience , and in terms of finding a level of uncertainty that they were able to tolerate when deciding about a sampling grid spacing~~Evaluation of the four communication methods was done through a questionnaire by eliciting

opinions of participants with experience in planning surveys, about the method's comprehensibility and effectiveness and the

25 sampling density that they would select based on that method. Our results show ~~that the approach that stakeholders favoured was offset correlation, and some approaches were~~ significant differences in how the participants responded to the methods, with the joint probability and implicit loss function approaches being not well understood~~(conditional probability and implicit loss function)~~, and offset correlation was the most understood. During feedback sessions, the ~~stakeholders~~ information users highlighted that they were more familiar with the concept of correlation, with a closed interval of [0,1] and this explains the

30 more consistent responses under this method. The offset correlation will likely be more useful to ~~stakeholders~~information users, with little or no statistical background, who are unable to express their requirements of information quality based on other measures of uncertainty.

1   Introduction

~~Micronutrient deficiencies (MND)are a widespread health problem in sub-Saharan Africa (Hurst et al., 2013; Joy et al., 2014; Phiri et al., 20~~

35 ~~. In the GeoNutrition project, it has been shown that concentration of micronutrients in staple crops and in soils vary spatially and so interventions to address the deficiencies should be based on spatial information (Gashu et al., 2021; Botoman et al., 2022) . In our previous studies (Chagumaira et al., 2021, 2022) we have shown that geostatistical predictions can help stakeholders to make decisions for interventions to address MND, by accounting for uncertainty in the supporting information which has been produced by statistical prediction from data and covariates.~~

40 ## 1.1   Mapping to support decisions: importance of spatial mapping

Spatial information is needed to support decisions at different spatial scales. Many approaches can be used to predict soil or grain properties at unsampled locations (e.g. geostatistical methods and machine learning algorithms). These methods make predictions based on a set of point observations configured on a systematic grid or spatial coverage sampling design. Geostatistical methods capture the spatial dependence by modelling the variation as an outcome of a random process (Webster, 2000)

45 , whereas machine learning methods do not entail a statistical assumption about the distribution of soil or grain property. Therefore, geostatistical methods offer an approach to sampling because they leverage on the statistical model that provides a basis for planning sampling given a statistical model.

Spatial information is ~~obtained from surveys~~ usually derived from field data obtained in surveys. These surveys have costs, for example, travel and logistics, staff costs, time for community engagement, management costs and analytical costs for

50 processing material collected in the field. The denser the sampling the higher the quality of the resulting information (in the sense that the uncertainty attached to spatial predictions is reduced). However, there are diminishing returns to increasing survey effort, and ~~in most cases, budgets for surveys are limited and~~ so there is an optimal survey effort where the marginal costs of the ~~stakeholders involved may have different set of questions to address and common ground is often difficult to establish. Often surveys efforts are constrained by budgets and we need a trade-off between sample effortand uncertainty. Again, this~~

survey match the marginal improvement in the resulting information (Lark et al., 2022).

The dependence of the quality of spatial information on survey effort has been studied by geostatisticians. In a geostatistical model the value of a variable at an unsampled location has a prediction distribution, conditional on the model and the data. The variance of this distribution, however, the kriging variance, is conditional on the model only and so can be calculated from the model for any posited set of observations. McBratney et al. (1981) showed how, given a variogram model, ordinary kriging variances could be computed at the cell centres of square sampling grids of different spacing. A plot of kriging variance against spacing could be used to select a sampling grid spacing if a target kriging variance can be specified. A technical challenge is how to obtain the variogram before sampling. One might undertake a reconnaissance survey (particularly when a large final survey is envisaged) to estimate the variogram and use a Bayesian approach to account for its uncertainty (Lark et al., 2017), use a variogram from a cognate area (Alemu et al., 2022), use an average variogram for the variable derived from published studies (Paterson et al., 2018), use a variogram elicited from experts (Truong et al., 2013) or use an adaptive sampling strategy with several phases in which the spatial model is the primary output from early phases (Marchant and Lark, 2006). The general approach of sampling design for ordinary kriging, which McBratney et al. (1981) developed can also be extended to the more general case of spatial prediction from a linear mixed model with spatially correlated random effects and fixed effects which include covariates such as measurements from remote sensors, variables derived from digital terrain models and factorial covariates such as soil maps (Brus and Heuvelink, 2007).

1.2 Communicating the uncertainty of spatial information from proposed survey designs

Despite this effort to address the statistical component of survey planning, the generation of measures of uncertainty for
90 particular proposed designs, there has not been a complementary effort on how these measures are understood by information users, such as survey sponsors, who might have the final responsibility of setting a survey budget, and so determining the ~~resulting uncertainty , could be used to find an appropriate sample density such that the information user is satisfied with the trade off between cost and the~~ quality of ~~information. However, kriging variance is not an accessible~~ the resulting information. Previous studies, including that by Chagumaira et al. (2021), have shown that non-statisticians commonly do not find the
95 kriging variance a meaningful measure of uncertainty ~~for many end users (Chagumaira et al., 2021), and~~ to interpret spatial predictions, so it is ~~likely that other methods of communicating the implications of sampling density on uncertainty would be better~~unlikely they would find it useful as a measure of the quality of survey outputs to balance against costs.

~~Kriging is unbiased, on the assumption of normality of the kriging errors, prediction intervals can be computed from kriging variances. Prediction intervals reflect the spatial variability of the variable and density of the samples (Webster and Oliver, 2007)~~
100 ~~. However, we know that prediction intervals are not preferred by end-users as a method of communicating uncertainty when making decisions, they find it easier to interpret measures of uncertainty tied to a particular decisions(Chagumaira et al., 2021)~~ ~~. For example, the probability that the value of a soil property at some location does not exceed a threshold, below which some intervention is needed. However,~~ In this study we worked with information user groups (soil science, agronomy, nutrition, and public health) to examine how they interpret measures of survey quality, and whether they regard them as suitable for guiding
105 a decision on the density of samples to be required for a survey. The measures we considered were all ones which could be derived from an initial variogram of the target variable, and we outline them briefly here, more detail is given in Appendix A (A1–A4).

1.3 Proposed methods for communicating information quality

We consider two measures derived from the kriging variance ~~and prediction interval give measures of uncertainty but do not~~
110 ~~contextualise then with respect to the decision being made. If the purpose of the survey is to support decisions about a certain intervention then it may be useful for the end-user to understand the implications of sampling in terms of the risk that the resulting information results in incorrect decisions in particular situations. This can be considered as a general, decision related uncertainty measure.~~

~~We may consider some sampling design (e.g. square grid) and a notional unsampled location, x_0, at which information is~~
115 ~~needed to support a decision. For a conservative measure of uncertainty, x_0, may be at a general location where uncertainty is largest e.g. at the centre of a square grid cell. We may then compute, over a specified distribution of~~ as measures of information quality. The first is the prediction interval, the interval which includes the unsampled value with some specified probability. Prediction intervals for surveys on grids of different spacing were proposed, in visual form which allowed the user to evaluate them relative, for example, to differences between critical values of the target variable ~~, the probability that the predicted value~~
120 ~~of the property will indicate that no intervention is required at x_0 conditional on the true value, $Z(x_0)$, indicating that actually an intervention is needed. That is to say, the probability that~~ for management purposes. The second measure was based on the joint probability that a location requires some intervention (because the surveyed variable exceeds or falls below a threshold)

and that ~~the spatial information will fail to indicate the correct decision at sites where a management or policy intervention is required. This probability will depend on the specific statistical model for the variable but also on sampling density. The conditional probabilities can then be used to make a decision about soil sampling, by selecting an appropriate grid spacing which limits the risk to acceptable level. Conditional probabilities have not been used in this way before, but there might be a way to tie uncertainty to a specific decision in a way which will help the stakeholder to understand its significance.~~ spatial prediction at that location indicates the contrary. This was proposed because we found that information users were generally receptive to presentations of uncertain information based on the probability that the mapped variable falls above or below a significant threshold (Chagumaira et al., 2021).

~~A further way to develop the decision-focussed approach to sample planning is to consider the costs of sampling and the costs resulting from uncertainty. This requires the data user's loss function~~ The third measure which we considered is based on value of information theory (Journel, 1984; Lark et al., 2022). It is the implicit loss function (Lark and Knights, 2015). A loss function ~~expresses the costs incurred resulting from using erroneous information to make a decision for an intervention (Goovaerts, 1997). The loss function determines the expected loss when the prediction~~ represents the loss incurred when a decision is based on spatial information which is correct (loss $=0$) or in error (loss ≤ 0). This is used to ~~make a decision. We can then compute the expected loss for a decision as a function of the precision of supporting information which in turn depends on the sampling density, and compare this with the cost of obtaining sample data with that density. It may not be possible to define a loss function prior to making decisions on soil sampling strategy because the cost of the errors are difficult to frame and quantify. However, an~~ analyse quality of information in cases where losses are reasonably straight forward to specify for different scenarios (e.g. Ramsey et al., 2002). Lark and Knights (2015) proposed that, for more complex cases, the implicit loss function might be used in critical assessment of a specified level of survey effort, based for example, on a fixed budget. An implicit loss function ~~, conditional on a logistical model (i.e. a function of sampling effort~~ is one which, given a model of survey logistics, and statistical information ~~about the estimates of the cost of errors) can be modelled as the loss function that makes a particular decision on sampling effort rational (Lark and Knights, 2015). The logistical model can be obtained from data from a previous survey or a from a comparable region. Lark and Knights (2015) suggested~~ (such as a variogram when the information is obtained by geostatistical prediction) makes a specified survey density the rational choice, i.e. the choice under which a marginal increase in survey cost is equal to the marginal reduction in expected loss when decisions are based on the resulting information. Lark and Knights (2015) proposed that reflection on the implicit loss function ~~for different sample schemes, or competing projects, may help decision-makers to arrive at loss functions which might regarded as plausible.~~

~~Decisions on soil sampling can be based on more general measures of uncertainty, that relate to sampling intensity, such as the offset correlation (Lark and Lapworth, 2013). The offset correlation~~ would help information users to decide whether a proposed survey budget is consistent with information users' views on the implications of making decisions with uncertain information, and we evaluated that here.

The fourth measure which we considered is the offset correlation. This is a measure of the ~~robustness of the resulting map to arbitrary variation in the location of the origin of a fixed regular sampling grid . For example,~~ consistency of spatial information produced when surveying at a particular grid spacing. Lark and Lapworth (2013) considered a hypothetical case

in which a variable is mapped by ordinary kriging from data on a sample grid of spacing ζ, a second map is then made of the same variable and from a grid of the same spacing, but in which the origin is shifted from the original grid by $\zeta/2$ in each

160   direction. They showed that, for a specified variogram, the offset correlation increases as the uncertainty in the map, attributable to sample density, decreases. It is not directly related to the decision process but dependent on the variogram and the proposed sampling spacing. The offset correlation might be a more intuitive uncertainty measure than prediction intervals and kriging variances. This is because people can more easily grasp and evaluate bounded measures such as the correlation (Hsee, 1998).

In this study we aimed to find out whether groups of stakeholders were able to make decisions on soil and crop sampling

165   strategies, in particular sampling density using soil pH and selenium concentration in grain (Se_{grain}), with the methods described above. We aimed to address the following questions: (i) *can stakeholders use the different approaches consistently?* (ii) *do the stakeholders have a preference?* and (iii) *does their use/preference depend on their background and experience?*

In the next section of this paper, we describe in detail the test approaches.

2   Theory

1.1   Prediction interval

Some unknown quantity at a location (e.g. soil pH or Se_{grain}) is characterised by a prediction distribution conditional on the data and statistical model. The kriging variance at the unsampled location , x_0, is defined as

$$\sigma_K^2 = E[\{Z(\mathbf{x}_0) - \tilde{Z}(\mathbf{x}_0)\}^2],$$

where $\tilde{Z}(\mathbf{x}_0)$ is a prediction of the random variable $Z(\mathbf{x}_0)$. The kriging prediction is a weighted average of the data

175   $$\tilde{Z}(\mathbf{x}_0) = \sum_{i=1}^{N} \lambda z(\mathbf{x}_0),$$

where $z(\mathbf{x}_0)$ is the data and λ are the kriging weights (Webster and Oliver, 2007). Then the kriging variance, σ_K^2 is given by:

$$\sigma_K^2 = E[\{Z(\mathbf{x}_0) - \tilde{Z}(\mathbf{x}_0)\}^2].$$

Cross-validation predictions of the statistical model need to be examined by exploratory analysis of correlation of the mapped

180   values at some location increased as the sampling grid became denser. We suggested that this minimum offset correlation (at a location furthest from a sample point in either grid) is an intuitive measure of the quality of a survey output, it shows the extent to which the mapped value of the kriging error, $\varepsilon(\mathbf{x}_0) = \{z(\mathbf{x}_0) - \tilde{Z}(\mathbf{x}_o)\}$ to check if the assumption of the normality holds. The kriging predictor is unbiased and the mean of the errors is zero, and their standard deviation is equal to the kriging

standard deviation, σ_{K}, from kriging. Based on this, a 95% prediction interval can be computed as: variable is robust to the location selected as the origin of the survey grid.

$$\left[\tilde{Z}(\mathbf{x}_0) - 1.96\sigma_{\mathrm{K}}(\mathbf{x}_0), \tilde{Z}(\mathbf{x}_0) + 1.96\sigma_{\mathrm{K}}(\mathbf{x}_0)\right].$$

The prediction distribution may also be obtained on a block support—for example if predictions are required at the scale of a farm mean or a mean for an administrative region. The same approach holds to the derivation of a prediction interval Our primary interest was how far information users felt confident in using these measures of the quality of spatial information as a basis for selecting the sample spacing (and hence the cost) of a hypothetical survey which they were engaged in planning. This is not a sufficient basis for deciding whether a criterion is robust and useful, but it is a necessary basis, since unless information users feel that they understand a method and its implications they cannot be expected to use it. We also examined how far the information users interpretations of the criteria were internally consistent (i.e. with the definitions of each criterion) and examined how far they resulted in consistent selections of grid spacing for a particular variable.

1.1 Conditional probability

We can calculate the joint probability that a location requires an intervention, and that the kriged estimate does not indicate this. If $\mathbf{x}_0$ is the location of interest, $\tilde{Z}(\mathbf{x}_o)$ is the prediction and $z(\mathbf{x}_0)$ the value of the variable at $\mathbf{x}_0$, then $\tilde{Z}(\mathbf{x}_o) - z(\mathbf{x}_0) = \varepsilon(\mathbf{x}_0)$, the error of the kriging predictions. The covariance of $z(\mathbf{x}_0)$ and $\varepsilon(\mathbf{x}_0)$ is:

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$$\mathrm{Cov}\left[z(\mathbf{x}_0), \varepsilon(\mathbf{x}_0)\right] = \mathrm{Var}\left[Z(\mathbf{x}_0)\right] - \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{c},$$

where $\boldsymbol{\lambda}$ denotes the vector of kriging weights for observations used to make the prediction, and $\mathbf{c}$ denotes the vector of covariances between each of these observations and $Z(\mathbf{x}_0)$. We can therefore , specify the joint distribution of $\{z(\mathbf{x}_0), \varepsilon(\mathbf{x}_0)\}$, assuming a normal random variable and prediction errors and conditional on the variance parameters of a geostatistical model. We also specify some $\mathbf{x}_0$ which will give a conservative output—e.g. for a square grid we could specify $\mathbf{x}_0$ at the centre of a grid cell where kriging variance is largest. From this it is possible to compute the conditional probability that $\tilde{Z}(\mathbf{x}_0) \geq z_t$ given that $z(\mathbf{x}_0) < z_t$, i.e. the probability, given that an intervention is required at $\mathbf{x}_0$ that, due to error in prediction, the mapped variable does not show this. A detailed description of how the desired conditional probability can be obtained from the joint probabilities is presented in the Supplement.

2.1 Implicit loss function

The loss function is a function of the error of $\tilde{Z}$, the kriging estimate of Z, as an estimate of the true unknown value, z, $\mathcal{L}(\tilde{Z} - z)$. The loss function is explained in greater detail by Journel (1984), Goovaerts (1997) and Lark and Knights (2015).

Journel (1984) defined a general linear loss function as:

$$\mathcal{L}(\tilde{Z} - z) = \alpha_1 |\tilde{Z} - z| \text{ if } \tilde{Z} < z$$

$$= \alpha_2 |\tilde{Z} - z| \text{ if } \tilde{Z} \geq z.$$

215 The parameters α_1 This study was conducted with information-users who have been involved with the GeoNutrition project (http://www.geonutrition.com/), which examined strategies to alleviate micronutrient deficiencies in Ethiopia and Malawi and included surveys to provide baseline information on micronutrient concentrations in staple crops and soils, and α_2 have positive real values. The coefficient α_2 is the loss per unit error of underestimation and α_1 is the loss per unit of error of overestimation. The slopes, α_1 and α_2 define the asymmetry of the loss function. The loss function can be symmetrical,

220 i.e. penalizing overestimation and underestimation equally; or can be asymmetrical because over-and-underestimation have different consequences. The asymmetry of the loss function is the ratio of the loss per unit value by which a quantity is underestimated to the loss per unit value of an overestimation (Lark and Knights, 2015). The asymmetry, a, is obtained by

$$a = \frac{\alpha_2}{\alpha_1},$$

i.e., is independent of the absolute value of z. If the loss function depends only on the estimation error, then z can

225 be set to zero, without loss of generality and the expected loss can be computed as a function of the error variancesoil properties (such as pH) which influence soil to plant transfers of micronutrient. The GeoNutrition project had teams from multiple disciplines (agriculture, soil science, human nutrition, and public health). It has been shown that concentrations of micronutrients in staple crops and in soils vary spatially, as do biomarkers for micronutrient status and so interventions to address the deficiencies should be based on spatial information on all these variables (Gashu et al., 2020; Botoman et al., 2022)

230 . The spatial information therefore must be interpreted by information users from this broad set of disciplines, and all of them might also contribute to decisions on the amount of effort to be expended on field survey. It is plausible that experts with training in different disciplines might find different quantitative methods to express uncertainty in information useful for decision-making, and so we recruited a panel for elicitation which spanned these disciplines. We recruited the panel from institutions which were partners of the GeoNutrition project research team and the allied Translating GeoNutrition project

235 in Zimbabwe (ZimGRTA) and the University of Zambia. The participants were volunteers, recruited from national-level institutions with responsibility for interventions and policy working on agricultural research and extension services, public health bodies and nutritional research. Soil scientists from the UK were also included. Panel members were invited by email from the local GeoNutrition/ZimGRTA lead. Most of the participants were familiar with the GeoNutrition project. They had an interest in contributing to the planning, execution, and interpretation of surveys to address micronutrient deficiencies, and

240 so of the sample size (Lark and Knights, 2015). Increasing sample size reduces the minimum expected loss in so far as it reduced the error variance. Therefore, the cost of obtaining n samples can be measured at which the marginal cost of additional sample point is equal to the reduction in expected loss that single sample achieves (Goovaerts, 1997). However, it maybe difficult to define a loss function prior to making decisions about sampling. The losses may not be easy to quantify, e.g.

~~social costs of failing to intervene, costs of unnecessary interventions, loss of confidence in the decision-making organisation.~~

~~Stakeholders can be helped to reflect on possible loss functions through the implicit loss function. It is a loss function that makes a specified sample size, n, a rational choice, given the marginal costs. That is to say, it is the loss function implied by a choice of $\bar{n}$, assuming this is rational. The implicit loss function is conditional on a logistic model, that expresses the marginal costs of sampling exercise and the conditional distribution of z as a function of effort (Lark and Knights, 2015) and is obtained by finding $\bar{\alpha}_1$ (given asymmetry), such that~~ were aware of the importance of being able to engage with the process. Ethical approval to conduct this study was granted by the University of Nottingham, School of Biosciences Research Ethics Committees (SBREC202122022FEO) and participants gave informed consent to their participation and subsequent use of their responses. No remuneration was offered, but all participants in African countries who were not able to participate from institutional offices were provided with a one-day data bundle to allow them to join online.

$$\check{\mathcal{L}}(\bar{n}-1|\bar{\alpha}_1,\bar{\alpha}_2,\phi) - \check{\mathcal{L}}(\bar{n}|\bar{\alpha}_1,\bar{\alpha}_2,\phi) = \mathrm{C}(\bar{n}) - \mathrm{C}(\bar{n}-1),$$

We sought to explore how the information from the sampling conducted in the GeoNutrition project could be used to support decisions about sampling density for other similar projects. We used data from the GeoNutrition project, crop and soil properties were measured at national scale in Malawi.

~~where $\bar{n}$ is the specified number of sample, $\mathrm{C}(n)$ is the function that returns the cost of n samples and ϕ is a vector of variogram parameters, so kriging variance is a contributor. The asymmetry can be set at different values, or inferred from other elicited opinions of the stakeholder group (Lark and Knights, 2015). The expected loss can be minimised at a location given some prediction distribution of $\tilde{Z}$ for the variable of interest by specifying the value of variable corresponding to a given probability (P_0), i.e.,~~

$$\tilde{Z} = F^{-1}(P_0).$$

~~Where, F^{-1} denote the quantile of the prediction distribution for a probability P_0 obtained from~~

$$P_0 = \frac{\alpha_2}{\alpha_1 + \alpha_2},$$

~~(Journel, 1984). Lark and Knights (2015) suggested that a stakeholder group might consider an implicit loss function for different $\bar{n}$ as starting points in the elicitation of a sample size, or compare implicit loss function for different projects given different partitions of a total budget between them. No attempt has been made to elicit opinions from stakeholders on implicit loss function, so we tried it in this study.~~

2.1 ~~Offset correlation~~

~~The measure of the correlation that is expected between the kriging predictions , $\tilde{Z}_1(\mathbf{x}_0)$, made from the first square grid, of interval ζ, and predictions, $\tilde{Z}_2(\mathbf{x}_0)$, made from the second grid, a translation of the first grid by $\zeta/2$ in both directions is known~~

~~as the offset correlation. The correlation of the two kriging predictions can be computed by:~~

$$\rho_{\tilde{Z}_1,\tilde{Z}_2} = \frac{\mathbf{C}_{\tilde{Z}_1,\tilde{Z}_2}(\mathbf{x}_0)}{\sqrt{\sigma^2_{K_{\tilde{Z}_1}}\sigma^2_{K_{\tilde{Z}_2}}}},$$

275 ~~where $\mathbf{C}_{\tilde{Z}_1,\tilde{Z}_2}(\mathbf{x}_0)$ is the covariance $\tilde{Z}_1(\mathbf{x}_0)$ and $\tilde{Z}_2(\mathbf{x}_0)$. $\sigma^2_{K_{\tilde{Z}_1}}$~~ In this study we used the data from a geostatistical survey conducted in Malawi for the GeoNutrition project (Gashu et al., 2021). In this survey, field sampling was undertaken to support the spatial prediction of micronutrient concentration in crops and soil across Malawi. The sampling design was selected to achieve spatial coverage and used 'main-site' and 'close-pair' sampling to support the estimation of variance parameters of the linear mixed model (Lark and Marchant, 2018). The location of sample points were the centroids of the Delauny polygons,

280 resulting from the stratification function in the spcosa library for the R platform (Walvoort et al., 2010). The sample support (0.1 ha circular plot) for the data consisted of bulk soil and grain samples from aliquots within a single field (Gashu et al., 2020). Therefore, the uncertainty quantification of the predictions relates to the mean values of the target variable across such as support within a field at a specified location, and this is appropriate for deriving regional sampling densities. A detailed description of soil and crop sampling in Malawi are presented by Gashu et al. (2021) and Botoman et al. (2022), ~~and $\sigma^2_{K_{\tilde{Z}_2}}$ are~~

285 ~~the kriging variances of the predictions from the first and second grid, respectively.~~ the full data description is provided by Kumssa et al. (2022). Grain and soil samples were prepared and analysed using methods described in Gashu et al. (2020) and Kumssa et al. (2022).

~~The offset correlation depends on $\mathbf{x}_0$, and is smallest at the location furthest from points on either grid. This minimum offset correlation is used to evaluate predictions from a grid spacing ζ. Offset correlation is bounded on the interval 0,1, which~~

290 ~~makes it intuitively easy to interpret as an uncertainty measure. As the uncertainty in the map, attributable to sample density, decreases, the offset correlation increases. The denser the grid the more consistent the maps and the offset correlation will be 1 if the maps are identical and 0 if they are entirely unrelated to each other. Lark and Lapworth (2013) describes the offset correlation in greater detail.~~

3 ~~Materials and methods~~

295 2.1 ~~Basic approach~~

~~We used the four methods , described above, to assess uncertainty in relation to sampling density, considering the problem of measuring a soil property relevant to crop management: soil pH, and a property of the crop: Se_{grain} concentration.~~ We used variograms ~~from a national survey in Malawi for each variable (Gashu et al., 2021)~~ for soil pH and grain Se (Se_{grain}) concentration to obtain sampling densities for further notional sampling for an administrative district in Malawi, Rumphi

300 District, with an area of ~~4,769 km²~~4769 km². The outputs were presented to participants ~~. The participants considered each method in turn and were asked to select a sampling grid density based on the method. After doing this they were asked, for each method: Has the method helped you assess the implication~~ in poster format through PowerPoint, and examples of the posters are shown in Figures S5 – S10 in the Supplement.

2.1 Format of the exercise

We wanted to elicit from stakeholder the usefulness of proposed methods (offset correlation, prediction intervals, joint probabilities) in helping them assess the implications of uncertainty in spatial prediction in as far as ~~it~~ this is controlled by sampling~~?They were then asked: Which of these methods was easiest to interpret?Finally, the participants were asked to rank the method in terms of ease of use. Evaluation of the test methods were done using an online questionnaire on Microsoft Forms~~, considering the problem of measuring a soil property and a micronutrient from a crop. Soil pH and Se_{grain} were used as examples for this case study. We invited professionals working in agriculture, nutrition and health at civic organisations, universities, government departments from Ethiopia, Malawi and wider GeoNutrition sites (United Kingdom, Zambia, and Zimbabwe). In total we had 26 participants (18 were agronomists or soil scientists and 8 public health or nutrition specialists).

The elicitation was conducted online using Zoom Video Communications (2022) in two sessions, 26th and 28th April 2022. There were two sessions in order to accommodate participants from different time zones, and to manage the participants in smaller groups to allow for questions and feedback. The invited participants self-identified as (i) agronomist or soil scientist or (ii) public health or nutrition specialists. The participants also self-assessed their statistical/mathematical background and their frequency of use of statistics in their job role (perpetual, regular ~~,~~and occasional use).

~~We invited professionals working in agriculture, nutrition and health at civic organisations, universities, government departments from Ethiopia, Malawi and wider GeoNutrition sites (United Kingdom, Zambia and Zimbabwe). In total we had 26 participants (18 were agronomist or soil scientist and 8 public health or nutrition specialists). Ethical approval to conduct this study was granted by the University of Nottingham, School of Biosciences Research Ethics Committees (SBREC202122022FEO) and participants gave informed consent to their participation and subsequent use of their responses.~~

~~In the~~ In the exercise, an introductory talk was given to explain the study's objectives. During the talk, we explained the four test methods (offset correlation, prediction intervals, ~~conditional probabilities~~joint probabilities, and implicit loss function) and how they can be used to assess the implications of uncertainty in spatial predictions to determine appropriate sampling grid space for a geostatistical survey. We explained the structure of the questionnaire to the participants. We emphasized to the participants that we were not testing their mathematical/statistical skills and understanding but rather were testing the accessibility of the methods~~using their answered responses~~.

The participants considered each method in turn and were asked to select a sampling grid density based on the method. Evaluation of the test methods was done through a questionnaire, as shown on ~~Figure 1.~~ Table 1. Using the first four questions, Q1 to Q4, we wanted to find out if the method helped to identify a sampling grid spacing. On Q5, we wanted the participants to assess the test methods in terms of their effectiveness in finding an appropriate grid spacing. We asked the participants to rank these methods in an order of their effectiveness, in their experience, and in terms of finding a level of uncertainty that they were able to tolerate when deciding about a sampling grid spacing. We asked them to put rank 1 as the most effective method and rank 4 the least.

Table 1. The list of questions used to elicit stakeholder opinions about the set of methods that can help end-users to assess the implications of uncertainty in spatial prediction in as far as this is controlled by sampling.

Number	Question	Responses
Q1	We show you here some pairs of example maps of soil pH/Se$_{grain}$, each pair being based on a different grid spacing, and we also show scatter plots which illustrate the strength of the correlation so with a different offset correlation. What do you think is the smallest correlation that would be acceptable if one of the maps were to be used to make decisions?	(1) 0.4 (2) 0.5 (3) 0.6 (4) 0.6 (5) 0.8 (6) 0.9
Q2	You are shown different scenarios for the prediction of soil pH/Se$_{grain}$ from different grid spacing's, which determine the width of the prediction interval. What is the grid spacing that gives the widest prediction interval that would be acceptable if one of the maps were to be used to make decisions?	(1) Spacing-20km (2) Spacing-40km (3) Spacing-60km (4) Spacing- 80km (5) Spacing-100km (6) Spacing-120km
Q3	At some location on the map the true value of soil pH/Se$_{grain}$ indicates that an intervention is required, due to error in prediction there is a non-zero probability that the mapped soil pH/Se$_{grain}$ does not show this, this probability increases with grid spacing as shown on the graph. What grid spacing do you think corresponds to the largest acceptable value of this probability?	(1) Spacing-20km (2) Spacing-40km (3) Spacing-60km (4) Spacing-80km (5) Spacing-100km (6) Spacing-120km
Q4	We have three specified implicit loss functions for predictions Se$_{grain}$ concentration over an area of 4,769 square kilometres (km^2) for a district/administrative region. With the implicit loss function we assume that the sample density is fixed (e.g. on budgetary grounds)	(1) Spacing-10km (2) Spacing-20km (3) Spacing-30km

The The participants recorded their responses using and online questionnaire on Microsoft Forms. The offset correlation was the first method presented to the participants. This was followed by prediction intervals and ~~conditional~~ joint probabilities. The implicit loss function was the final method presented to the participants. We started with a measure we thought all our ~~stakeholder~~ stakeholders would most easily understand and then moved on to the more complex methods.

~~The list of questions used to elicit stakeholder opinions about the set of methods that can help end-users to assess the implications of uncertainty in spatial prediction in as far as this is controlled by sampling.~~

2.2   Test Methods

2.2.1   Statistical modelling and spatial prediction of grain Se concentration and soil pH

We ~~used the data from a geostatistical survey conducted in Malawi for the GeoNutrition project (Gashu et al., 2021). Field sampling was undertaken to support the spatial prediction of micronutrient concentration in crops and soil across Malawi. Detailed description of soil and crop sampling in Malawi are presented by Gashu et al. (2021) and Botoman et al. (2022), and the full data description is provided by Kumssa et al. (2022).~~

~~We~~ undertook exploratory analysis of soil pH and Se_{grain} concentration using QQ plots, histograms and summary statistics to check whether there was need for transformation of the variables for the assumption of normality. The data for Se_{grain} concentration were skewed and it was necessary to transform them to natural logarithms. The variance parameters for both soil pH and Se_{grain} concentration were estimated by residual maximum likelihood using the likfit procedure in the geoR packages (Diggle and Ribeiro, 2010) for the R platform (R Core Team, 2022) with a constant mean as the only fixed effect. These variance parameters were used in the subsequent test methods. The thresholds we considered, in this study for the prediction intervals and ~~conditional~~ joint probabilities were soil pH of 5 and Se_{grain} concentration of 38 µg kg^{-1}. The threshold for soil pH is 5 in Malawi, such that if the pH at a location falls below 5, it would be necessary to apply lime (Chilimba et al., 2013). The threshold Se_{grain} concentration is 38 µg kg^{-1}, such that a serving of 330g of grain flour provides a third of the daily estimated average requirement of ~~Se_{grain}~~ Se for an adult woman (Chagumaira et al., 2021). The intervention for soil pH was liming, and Se_{grain} was provision of fortified food.

2.2.2   Offset Correlation

We presented the participants with correlated pairs of hypothetical maps of soil pH and Se_{grain}, with differing correlations, so that the extent to which maps might differ as a result of the grid offset could be visualized. We also showed scatter plots that illustrated the strength of the correlation. Figure 1, shows an example of pair of maps of soil pH and the corresponding scatterplot (see also Figures S5 and S6). The correlation plots showed the kriging predictions for soil pH or Se_{grain} concentration predicted with parameters estimated in Section 2.2.1. We asked the participants the smallest offset correlation that would be acceptable if one of the maps were to be used to make decisions based on the soil or grain property. Neither of the posited pair of maps, based on offset grids, is to be regarded as closer to reality than the other, the question is how consistent they are.

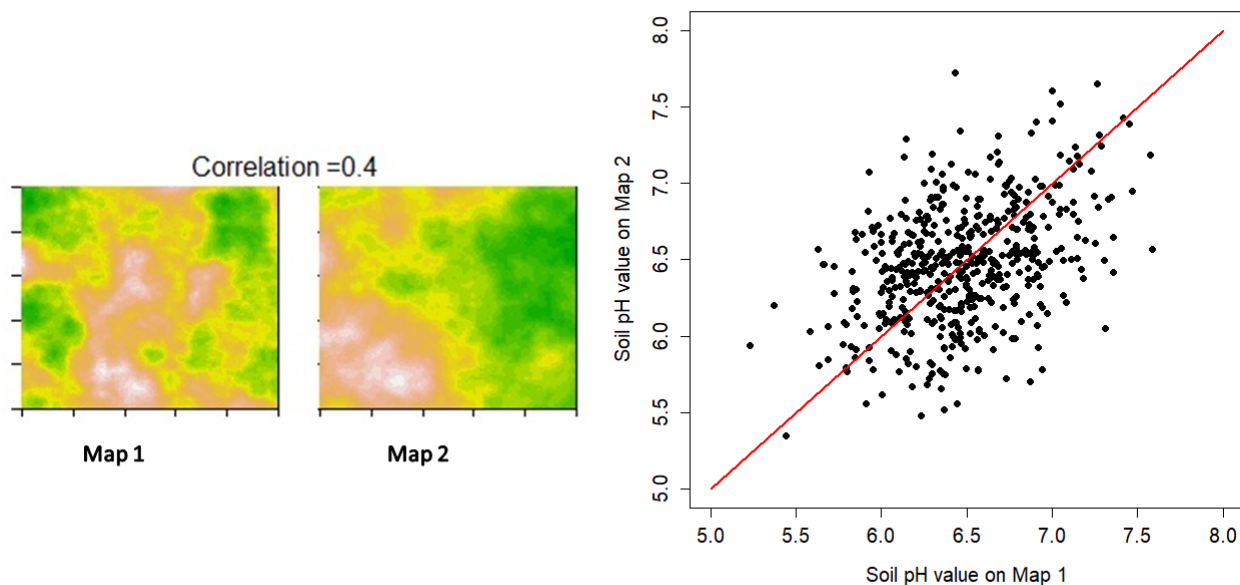



Figure 1. The pair of hypothetical maps of pH value and corresponding scatter-plot for offset correlation 0.4.

2.2.3 Prediction intervals

Using the variance parameters estimated in Section 2.2.1, we evaluated kriging variances at the centres of cells of square

370 grids of different spacings. We considered minimum and maximum grid spacings of 0.05 and 125 km, respectively, with an increment of 0.5 km. We then computed the cell-centred block kriging variance the spacings we were considering by block kriging (Webster and Oliver, 2007). ~~We considered different prediction~~ For all the grid spacings, we computed cell-centred block kriging variance on 0.01 km² square blocks. We considered three different predictions for each variable but the prediction interval was fixed, depending only on grid spacing. The three predictions of soil pH were 4.8, 5.5 and 6.0 and those of Se_{grain}

375 were 20, 55 and 90 μg kg^{-1}. The predictions of soil pH and Se_{grain} concentration were presented to the participants in a chart as shown in Figure 2.

The chart consisted of (a) box plot of the distribution of the measured variable based on all soil samples from the study area, (b) a graph of the lower and upper prediction intervals for the prediction at the point of interest for grid spacings from 0 to 120 km, and lines indicating (c) the z_t and (d) the prediction (see ~~Figure 2a~~Figures 2, S7 and S8). From the chart, we

380 asked the participants to select the grid spacing that gives the widest prediction interval that would be acceptable if the mapped predictions were to be used to make decisions about soil management or interventions to address human Se deficiency.

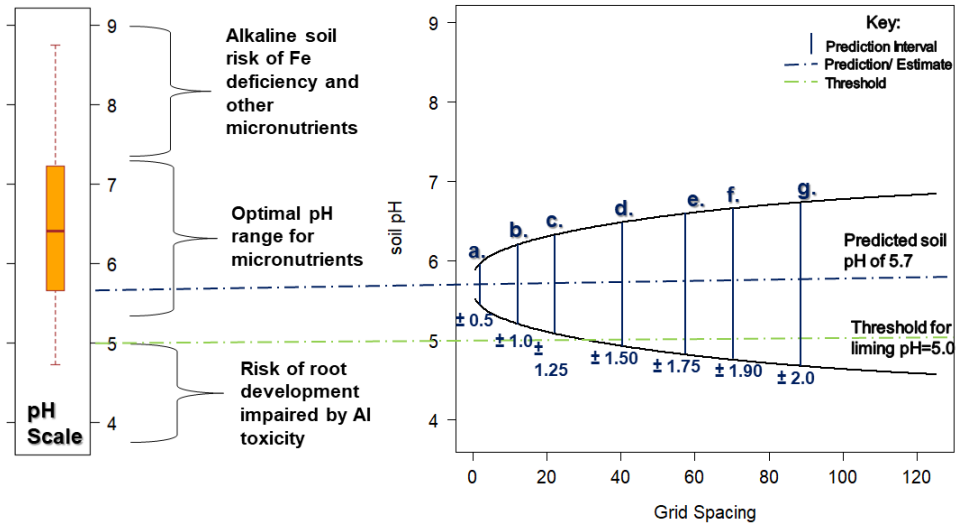



Figure 2. An example of a chart, for prediction intervals, with prediction of soil pH of 5.7 with prediction intervals and in relation to a threshold of pH = 5.0.

2.2.4 ~~Conditional~~ Joint probability

The ~~conditional~~ joint probability is a measure of uncertainty in terms of the risk of failing to intervene at some location given that an intervention is needed. We presented the participants with a chart of ~~conditional~~ joint probabilities plotted against grid spacing is shown on Figure ~~2b and~~ 3 (see also Figure S9), and this probability increases with grid spacing. The ~~conditional~~ joint probability is bounded on an interval [0,1]. A probability of 1, indicates that the prediction will be equivalent to the overall mean of the dataset. If the prediction of Se_{grain} or pH was below the threshold, z_t, an intervention is needed. We then asked the participant ~~at~~ what grid spacing they thought corresponded to the largest acceptable value of this probability.

2.2.5 Implicit loss functions

In order to compute the implicit loss function, we needed a cost model for Rumphi district. We used the function defined in Lark and Knights (2015) to return the costs of n samples over an area A km^2, i.e. a sample density of $r = N/A$ samples per km^2 :

$$C(n) = \omega + vAr + \beta At_r, \tag{1}$$

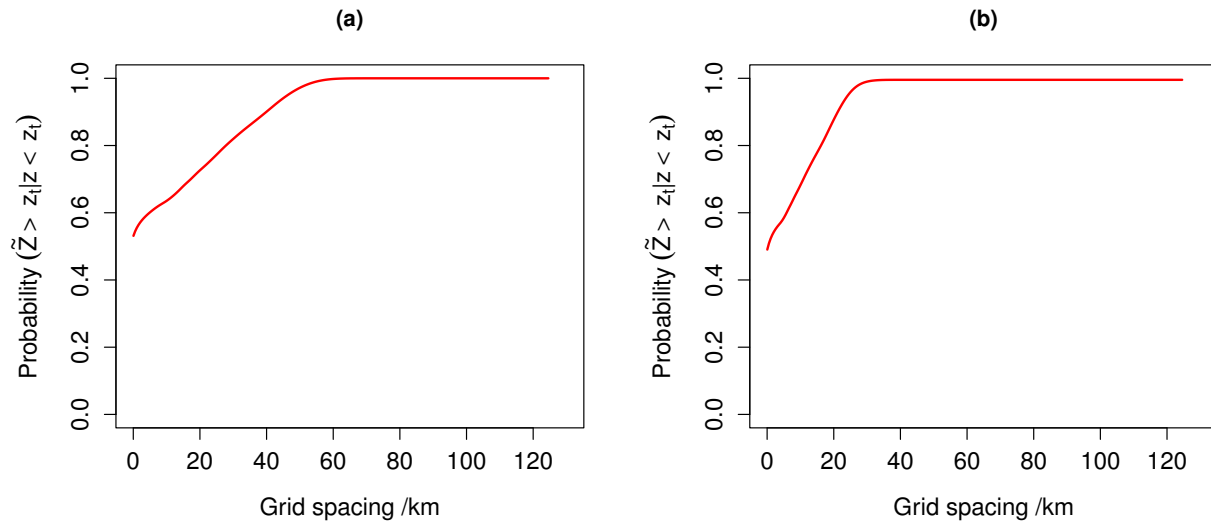



Figure 3. An example of the chart of ~~conditional~~ joint probabilities plotted against grid spacing for (a) soil pH and (b) Se_{grain} concentration. At a location $\mathbf{x}_0$, $\tilde{Z}$ is the prediction and z is the value of the variable at that location.

where ω are the fixed costs, υ cost of laboratory analysis per unit, and β the field costs per work day per team. The quantity t_r is time taken to sample per km^2 at a density of r per km^2. We obtained these costs for Rumphi district by considering the available costs for crop sampling during the GeoNutrition survey conducted in Malawi at national-scale (Gashu et al., 2021; Kumssa et al., 2022). A detailed description of how the costs were computed is presented in the Supplementary Material.

We fixed the asymmetry ratio ~~as~~ at 1.5, ~~by~~ assuming the elicited mean probability threshold from similar stakeholders in Ethiopia and Malawi (Chagumaira et al., 2022) can be regarded as an approximation of P_0 which corresponds to a quantile of prediction distribution. This implied a bigger loss for overestimation of the variables (i.e. failing to intervene of Se_{grain} are smaller than prediction). With the implicit loss function we assumed that the sample density is fixed (e.g. on budgetary grounds) and computed the loss function which would make that a rational choice. We presented three specified implicit loss functions for predictions of Se_{grain} for Rumphi district, with an area of 4,769 km^2 with sampling densities fixed at 10, 20 and 40km. Figure ~~2c and~~ 4 (see also Figure S10), shows the implicit loss function for Se_{grain}. We then asked the participants to identify the loss function implied by the sampling decision that looked more plausible to make decisions about interventions to address human Se deficiency.

~~An example of specified implicit loss functions for predictions of Se_{grain} concentration at a 10km grid spacing.~~

2.3 Data Analysis

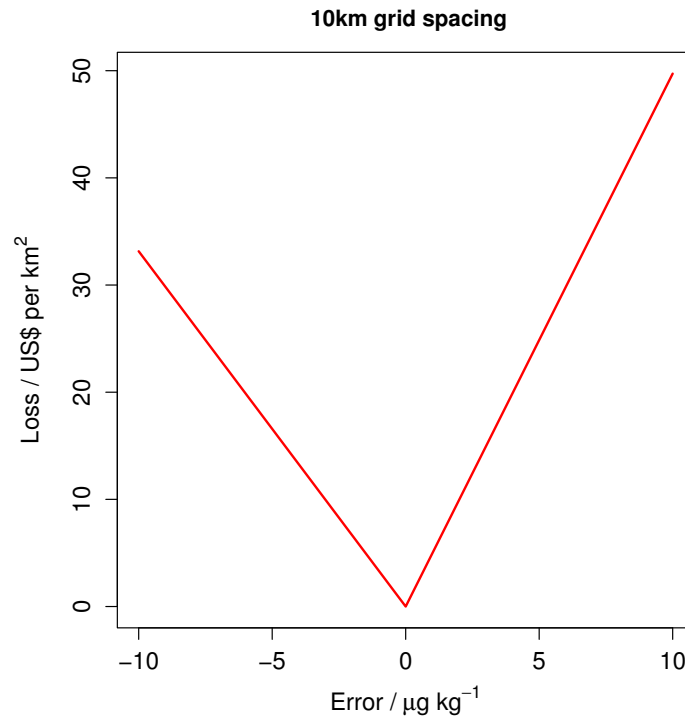
2.3.1 ~~Offset Correlation~~ Test methods

10km grid spacing

Figure 4. An example of specified implicit loss functions for predictions of Se$_{grain}$ concentration at a 10km grid spacing.

410 ~~We presented the participants with pairs of example maps of soil pH and Se$_{grain}$ concentration, each pair being based on a different grid spacing, and so with different offset correlation. We also showed scatter plots that illustrated the strength of the correlation. Figure 2d, shows an example of pairs of maps of Se$_{grain}$ concentration and the corresponding scatterplot (see Figure S5 and S6). The correlation plots showed the kriging predictions for soil pH and Se$_{grain}$ concentration predicted with parameters estimated in Section 2.2.1. We asked the participants the smallest offset correlation that would be acceptable if one~~

415 ~~of the maps were to be used to make decisions based on the soil or grain property.~~

~~The pairs of example maps of Se$_{grain}$ concentration and corresponding scatterplot for offset correlation 0.4.~~

2.4 ~~Data Analysis~~

2.3.1 ~~Test methods~~

~~The results for~~ The responses for Q1 to Q4 ~~were presented as contingency tables. The~~ were presented as contingency tables.

420 The contingency table allowed us to evaluate whether there were differences in the responses of the participants based on (i) variable used in the test method, (ii) professional group and (iii) by frequency of use of statistics. We analysed the contingency table on the basis of a null hypothesis that the distribution of observations between responses (e.g. selected grid spacing) is

independent of the factor in the column (e.g. professional group). If evidence is provided to reject the null hypothesis, then this would indicate that how a respondent interprets the information presented to select a grid spacing depends on their professional group. A detailed description of how we used the contingency tables to partition the responses of the questionnaire are presented in Appendix B.

The rows of each table correspond to the response (e.g. the different grid spacings) and, the full table, the columns correspond to the frequency of use of statistics, nested, within professional group and nested within variable used (soil pH or Se_{grain}). Contingency tables allowed us to test the null hypothesis of random association of responses with the different factors in the columns. The expected number of responses under the null hypothesis, $e_{i,j}$ in a cell $[i,j]$, is a product of row (n_i) and column (n_j) totals dived by the total number of responses (N), and this the null hypothesis of the contingency table which is equivalent to an additive log-linear model of the table. An alternative to the additive model for the contingency table, is the saturated model that has an extra $(n_r - 1)(n_c - 1)$ term that allows for interaction amongst the columns and tables of the table. The proportions of observed responses $o_{i,j}$ may differ from $e_{i,j}$ in a cell $[i,j]$ and the likelihood ratio statistic or deviance, L, can be used to provide evidence against the null hypothesis. The likelihood ratio statistic is computed by

$$L = 2 \sum_{i=1} \sum_{j=1} o_{i,j} \log \frac{o_{i,j}}{e_{i,j}}. \tag{2}$$

where L has an approximate χ^2 distribution under the null hypothesis of random association between the rows and columns of the table, with $(n_r - 1)(n_c - 1)$ degrees of freedom (Christensen, 1996; Lawal, 2014). We fitted the log-linear models using the loglm function from the MASS package (Venables and Ripley, 2002) for the R platform.

A contingency table can be partitioned to evaluate whether there are differences in the responses of the participants based on (i) variable used in the test method, (ii) professional group and (iii) by frequency of use of statistics. In Figure 3, we illustrate how the contingency table can be partitioned. The table can be partition into components corresponding to pooled table and subtables of the full table.

An illustration of how the log-likelihood ratio was used to partition full table into subtables and pooled tables.

The full table in Figure 3, was partitioned into subtables for soil pH (Subtable 1 in Figure 3), and Se_{grain} concentration (Subtable 2 in Figure 3). Then the pooled table completes the partition. The degrees of freedom and deviances for the three table sum to the degrees of freedom and deviance of the full table. Using the contingency table, we could conclude if there are differences in responses for the two variables. The full table in can further be partitioned, in a similar way, by the background of the respondents i.e., professional group and frequency of use of statistics.

In our study, we wanted to find out if the responses recorded by the stakeholders depended on the variable used (soil pH or Se_{grain} concentration), and background of the respondent. We expected the responses to differ. We thought the stakeholders would have different perceptions of the impacts of the uncertainty for soil pH and Se_{grain} concentration. There were more agronomist or soil scientists than public health or nutrition specialists in the meeting, and we expected the priorities of the groups to differ when making interventions for soil pH and Se_{grain} concentration. We also thought the frequency of use of statistics would influence the choice of method used to select an appropriate grid spacing.

We first tested for differences ~~in~~ responses recorded for each test method, by the variable used (soil pH or Se_{grain} concentration) using contingency tables. The responses from stakeholders in different professional groups were pooled within the two variables, as illustrated by the Pooled table 1 on ~~Figure 3.~~ Table 2. This gave us a six (responses) by two (variables) contingency table with 5 degrees of freedom for the questions corresponding to offset correlation, prediction intervals and ~~conditional~~ joint probabilities (Q1 to Q3). However, for the implicit loss function we did not consider this because we only had a loss function for Se_{grain} concentration.

Second, we considered if the differences in the responses depended on the professional group of the respondent. Finally, we considered whether the frequency of use of statistics in their job role had an impact on the responses recorded by the respondents. For some questions, we noted ~~differences~~ difference in the responses when pooled within variable used (soil pH or Se_{grain} concentration) and there was no differences in responses in professional groups and frequency of use of statistics for all questions. We further analysed the pooled tables or separate subtables to examine if the responses ~~where~~ were uniformly distributed and the null hypothesis is a random distribution. We wanted to test whether the responses of the participants were uniform, i.e., each grid spacing has equal likelihood of occurrence.

2.3.1 Assessment of the method

The responses for the Q5 were tabulated with the methods as the columns and ranks as the rows. The participants ranked their preferred method first. However, ~~in our analysis we reversed the order by assigning~~ to calculate the mean rank, $\bar{r}_i$, for each method for all the respondents, we assigned a score of 4 for the most preferred method and ~~1~~ 1 for the least. We computed the ~~mean ranks,~~ $\bar{r}_i$, for each method for all respondents. We then separated the respondents by their professional group and computed the mean ranks.

Finally we separated the respondents by their frequency of use of statistics in their job role. Under a null hypothesis of random ranking for set of k ranks, the expected mean rank is $(k+1)/2$. The evidence against this hypothesis is measured a statistic distributed as $\chi^2(k-1)$:

$$\frac{12n}{k(k+1)} \sum_{i=1}^{k} \left\{ \bar{r}_i - \frac{k+1}{2} \right\}, \tag{3}$$

where n is the total number of rankings (Marden, 1996).

3 Results

3.1 Test methods

3.1.1 ~~Method 1:~~ Offset correlation

The full table for Q1 and the subsequent subtables are presented in the Appendix of the paper (Tables ~~A1–A3~~ B2–B4). There were no differences in the responses of the when the columns were pooled by the variable used, soil pH vs. Se_{grain} concentration, $p = 0.656$ (Table ~~1).~~ 2). There were no differences in the responses when the columns were also pooled within professional

Table 2. Analysis of the question on offset correlation, Q1, according to variable used, professional group and frequency of use of statistics.

	Deviance ~~Degrees of~~ P(L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	54.57	55	0.491
Pooled by variable used (pH v. Se$_{grain}$)	3.29	5	0.656
Pooled by professional group	6.50	5	0.260
Pooled by frequency of use of statistics	8.35	10	0.595
Subtable–pooled counts: variable used			
Soil pH	27.01	25	0.352
Se$_{grain}$	24.2	25	0.507
Subtable–pooled counts: professional group			
Agronomist or soil scientist	26.25	25	0.394
Public health or nutrition specialist	21.81	25	0.646
Subtable–pooled counts: frequency of use of statistics			
Perpetual use of statistics	8.99	15	0.878
Occasional use of statistics	18.17	15	0.254
Regular use of statistics	19.06	15	0.211
Subtable–pooled counts			
Responses are uniformly distributed	17.69	5	0.003

~~There were no differences in the responses when the columns were also pooled within professional groups ($p = 0.491$) and frequency of use of statistics ($p = 0.595$). Further analysis of the question on offset correlation was based on~~ on pooled counts, see Table ~~A3~~B4. There was strong evidence to reject the null hypothesis that the responses are uniformly distributed ($p = 0.003$). Figure ~~4~~5 shows the responses of how all the participants responded to Q1, for offset correlation.

Most of the respondents selected offset correlation of 0.7 as the smallest offset correlation that would be acceptable if one of the maps were to be used to make decisions based on the soil or grain property. We extracted the grid spacings, for soil pH and Se$_{grain}$, corresponding to the selected offset correlation of 0.7. The spacings were extracted from a plot of offset correlation against grid spacing obtained from the variance parameters of each variable (see Figure S4). The grid spacing for soil pH is 25 km and for Se$_{grain}$ is 12.5 km. The grid spacing corresponding to the offset correlation for each variable were computed from the variogram of the variable.

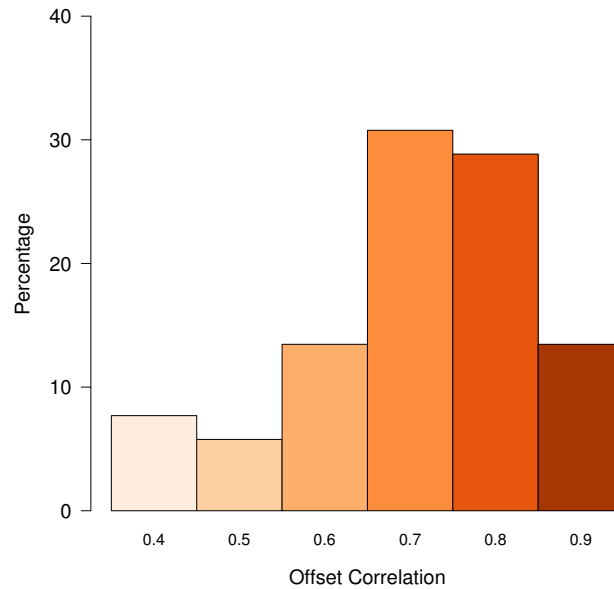



Figure 5. Bar charts showing how the participants responded to Q1 for offset correlation.

3.1.2 Prediction interval

There were no differences in the responses when pooled within the variable used, $p = 0.656$, for prediction intervals~~(Table 2),~~ see Table 3. We then pooled the responses within the professional groups, and there was no evidence to reject the null hypothesis ($p = 0.498$). Also, there were differences when responses were pooled within frequency of use of statistics, $p = 0.152$. Therefore, further analysis of the question on prediction intervals was based on pooled counts of responses. There was no evidence to reject the null hypothesis that the responses are uniformly distributed ($p = 0.169$). Figure ~~5~~ 6 shows the bar charts of how all the participants responded to the Q2 for prediction intervals. For this method, there no clear choice of grid spacing for sampling for soil pH and Se_{grain}.

3.1.3 ~~Conditional~~ Joint probabilities

Table ~~3~~ 4 shows the results for partitioning the contingency table for the question on ~~conditional~~ joint probabilities, Q3. There was strong evidence to reject the null hypothesis when the columns were pooled by variable used, $p \leq 0.001$. Therefore, further analysis was based on separate subtables for soil pH and Se_{grain} concentration. For both variables, there were no differences in the responses when the columns were pooled within professional groups and frequency of use of statistics. For soil pH there was strong evidence to reject the null hypothesis that the responses are uniformly distributed ($p \leq 0.001$). A similar result was found for Se_{grain} concentration ($p \leq 0.001$). The bar charts for the responses for the question on ~~conditional~~ joint probabilities

Table 3. Analysis of the question on prediction interval, Q2, according to variable used, professional group and frequency of use of statistics.

	Deviance ~~Degrees of P~~(L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	56.0	55	0.437
Pooled by variable used (pH v. Se_{grain})	0.972	5	0.965
Pooled by professional group	4.36	5	0.498
Pooled by frequency of use of statistics	14.5	10	0.152
Subtable–pooled counts: variable used			
Soil pH	23.8	25	0.531
Se_{grain}	31.2	25	0.181
Subtable–pooled counts: professional group			
Agronomist or soil scientist	26.5	25	0.381
Public health or nutrition specialist	25.1	25	0.455
Subtable- pooled counts: frequency of use of statistics			
Perpetual use of statistics	9.68	15	0.840
Occasional use of statistics	16.88	15	0.330
Regular use of statistics	15.08	15	0.450
Subtable- pooled counts			
Responses are uniformly distributed	7.77	5	0.169

for soil pH are presented in Figure ~~6a~~7a. The grid spacing chosen by the participants for soil pH is 60 km. The responses for Se_{grain} concentration are presented in Figure ~~6b~~7b. The grid spacing made was 40 km.

3.1.4 Implicit loss functions

The results for partitioning the contingency table for implicit loss function, Q4, are presented in Table 8. There were no differences in the responses when the columns of the table were pooled within professional groups ($p = 0.781$) and frequency of use of statistics ($p = 0.828$). Further analysis of the question on implicit loss function was based on pooled counts of responses. There was strong evidence to reject the null hypothesis that the responses are uniformly distributed ($p \leq 0.001$). The bar charts for the responses pooled counts for all respondents are shown on Figure 8. The grid spacing chosen by the participants for Se_{grain} concentration is 20 km.

~~Analysis of Q5 according to professional group and level of use of statistics in job role Test Statistic Degrees $P*(X^2)$of FreedomAll respondents61.13< 0.001~~

~~Professional groupAgronomist or soil scientist 493< 0.001Public health or nutrition specialist 15.63< 0.001~~

Table 4. Analysis of the question on ~~conditional~~ joint probabilities, Q3, according to variable used, professional group and frequency of use of statistics.

	Deviance ~~Degrees of P~~(L^2)	Degrees of freedom	P
Full contingency table analysis			
Full table	60.6	55	0.281
Pooled by variable used (pH v. Se$_{grain}$)	26.7	5	< 0.001
Pooled by professional group	5.32	5	0.378
Pooled by frequency of use of statistics	14.5	10	0.152
Subtable–pooled counts: variable used			
Soil pH	12.1	25	0.986
Se$_{grain}$	21.8	25	0.647
Soil pH subtable–pooled counts: professional group			
Pooled within professional group	4.48	5	0.483
Agronomist or soil scientist	3.10	10	0.979
Public health or nutrition specialist	4.50	10	0.922
Soil pH subtable–pooled counts: frequency of use of statistics			
Pooled within frequency of use of statistics	0.889	10	1.00
Perpetual use of statistics	4.50	5	0.480
Occasional use of statistics	4.36	5	0.499
Regular use of statistics	2.33	5	0.802
Soil pH subtable–pooled counts			
Responses are uniformly distributed	50.15	5	< 0.001
Se$_{grain}$ subtable–pooled counts: professional group			
Pooled within professional group	4.77	5	0.445
Agronomist or soil scientist	11.0	10	0.361
Public health or nutrition specialist	6.09	10	0.808
Se$_{grain}$ subtable–pooled counts: frequency of use of statistics			
Pooled within frequency of use of statistics	9.55	10	0.481
Perpetual use of statistics	1.73	5	0.886
Occasional use of statistics	5.55	5	0.353
Regular use of statistics	4.99	5	0.417
Se$_{grain}$ subtable–pooled counts			
Responses are uniformly distributed	36.77	5	< 0.001

Figure 6. Bar charts showing how the participants responded to the Q2 for prediction intervals.

3.1.5 ~~Implicit loss functions~~

~~The results for partitioning the contingency table for implicit loss function, Q4, are presented in Table 4. There were no differences in the responses when the columns of the table were pooled within professional groups ($p = 0.781$) and frequency of use of statistics ($p = 0.828$). Further analysis of the question on implicit loss function was based on pooled counts of responses. There was strong evidence to reject the null hypothesis that the responses are uniformly distributed ($p \leq 0.001$). The bar charts for the responses pooled counts for all respondents are shown on Figure 7. The grid spacing chosen by the participants for Se_{grain} concentration is 20 km.~~

3.2 Assessment of the test methods

The question on ranking of the method was analysed in three ways. At first we computed the mean ranks for all participants and tested for the evidence against the null hypothesis of random ranking. There is strong evidence to reject the null hypothesis of random ranking, $p \leq 0.001$ (Table ~~5~~6). Then the mean ranks were computed for each professional group and there was strong evidence to reject the null hypothesis of random ranking in each group ($p \leq 0.001$). Finally, we separated the participants into

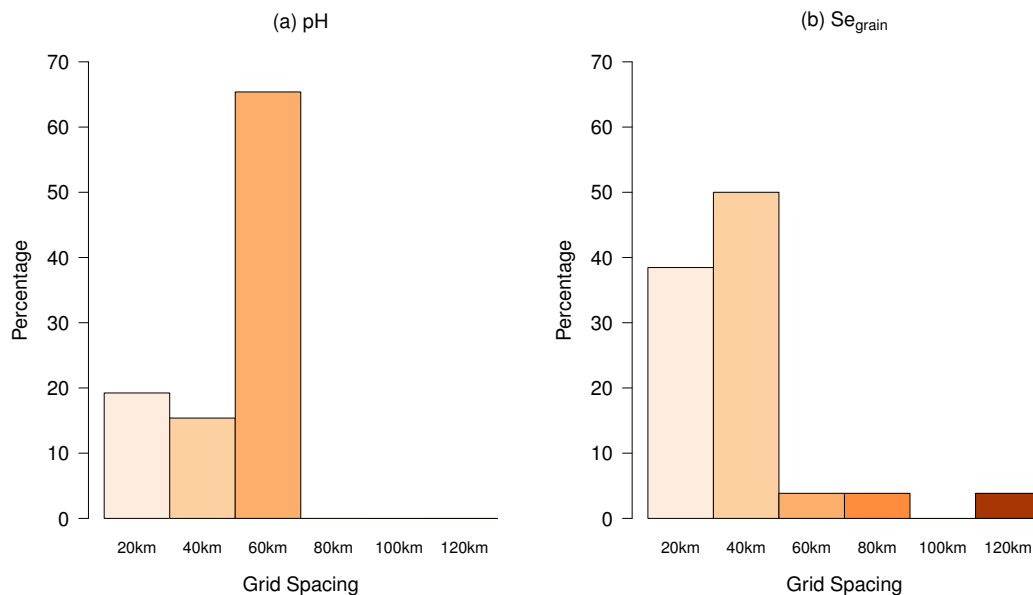



Figure 7. Bar charts showing how all the participants responded to the Q3 for ~~conditional~~ joint probabilities for (a) soil pH and (b) Se_{grain} concentration.

three groups according to their frequency of use of statistics in the job role, and computed the mean ranks in each group. There was strong evidence to reject the null hypothesis of random ranking in each group ($p \leq 0.001$).

~~Ranking of test methods in terms on the most effective: (a) by all respondents, professional group: (b) agronomists or soil scientist and (c) public health or nutritionist specialists, and frequency of use of statistics: (d) occasional use, (e) regular use and (f) perpetual use.~~

~~The~~ Amongst all the respondents, the offset correlation was ranked as the most effective ~~by all respondents~~ (Figure ~~8a~~9a) and implicit loss function as the least effective. Both professional groups (i.e. agronomist or soil scientist and public health or nutritionist) ranked offset correlation first but differed in the second and least ranked methods (Figure ~~8b to8c~~9b to 9c). Public health or nutrition specialists ranked second prediction intervals and implicit loss function as the least effective. The agronomist or soil scientist group ranked prediction intervals as the least effective and ~~conditional~~ joint probabilities as second.

When respondents were separated by their frequency of use of statistics, offset correlation was also ranked first (Figure ~~8d to8f~~9d to 9f). Those who use statistics occasionally, in their job role, ranked the implicit loss function as the second best and the prediction intervals the least. ~~Conditional~~ joint probabilities were ranked second and implicit loss function as the least effective by those who regularly use statistics in the job role. Those who ~~use statistic at all times~~always use statistics, ranked conditional probabilities second. Prediction intervals and implicit loss functions were ranked last.

Table 5. Analysis of the question on implicit loss function, Q4, according to variable used, professional group and frequency of use of statistics.

	Deviance $P(L^2)$	Degrees of freedom	P
Full contingency table analysis			
Full table	8.91	10	0.541
Pooled by professional group	0.49	2	0.781
Pooled by frequency of use of statistics	1.49	4	0.828
Subtable–pooled counts: professional group			
Agronomist or soil scientist	2.33	4	0.676
Public health or nutrition specialist	6.09	4	0.193
Subtable- pooled counts: frequency of use of statistics			
Perpetual use of statistics	1.73	2	0.422
Occasional use of statistics	1.73	2	0.422
Regular use of statistics	3.96	2	0.138
Subtable- pooled counts			
Responses are uniformly distributed	54.00	2	< 0.001

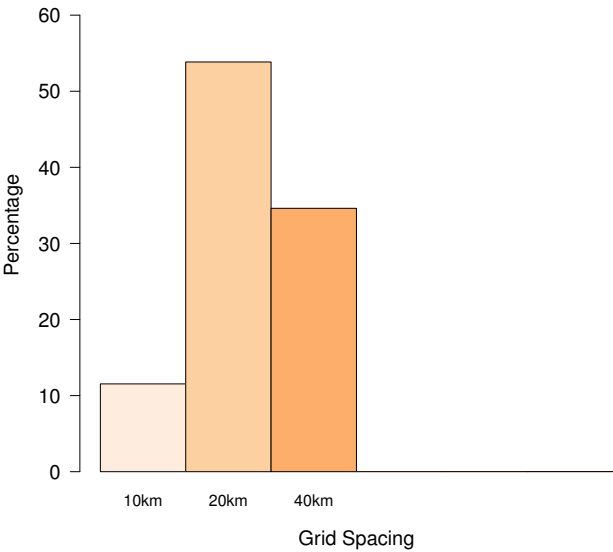



Figure 8. Bar charts showing how all the participants responded to the Q4 for implicit loss function.

Table 6. Analysis of Q5 according to professional group and level of use of statistics in job role

	Test Statistic (X^2)	Degrees of freedom	P^*
All respondents	61.1	3	< 0.001
Professional group			
Agronomist or soil scientist	49	3	< 0.001
Public health or nutrition specialist	15.6	3	< 0.001
Frequency of use of statistics			
Perpetual user of statistics	34	3	< 0.001
Occasional user of statistics	28.5	3	< 0.001
Regular user of statistics	49.8	3	< 0.001

4 Discussion

555 In this study, we presented to ~~diverse groups of stakeholders~~groups of information users, four methods (offset correlation, prediction intervals, ~~conditional~~ joint probabilities and implicit loss functions) that can be used to support decisions on sampling grid spacing for a survey of soil pH and Se$_{\text{grain}}$.

We wanted to find out if the ~~stakeholders~~ information users had a preference among the approaches presented to them. Offset correlation was ranked first as the method the stakeholders found easy to interpret (see Figure ~~8~~9), and ~~over 70%of the~~

560 ~~stakeholders specified a~~ most respondents (30 %) selected an offset correlation of 0.7~~or more as a criteria for adequate sampling intensity~~, and slightly fewer selected 0.8 so over half of respondents are accommodated within this range of values. During the feedback session, ~~stakeholders~~ information users highlighted that they were more familiar with the concept of correlation, with a closed interval of [0,1]. This ~~explains why there more consistent responses under this method~~is likely to explain the consistency of the results for this criterion, with over half the respondents selection 0.7 or 0.8 as a minimum acceptable

565 correlation. Our results are consistent with findings of Hsee (1998), that relative measures of some uncertain quantity (Hsee gives an example of the size of a food serving relative to its container) are more readily evaluated than absolute measures (the size of serving). An easy-to-evaluate attribute, such as the bounded correlation of [0,1], has a greater impact on a person's judgement of utility. Hsee (1998) describe this as the "relation-to-reference" attribute. It is therefore, not surprising that the offset correlation is highly-ranked.

570 The offset correlation ~~will be more useful for stakeholders who are not able to express their quality requirement for information in terms of quantities~~ seems to be a criterion which respondents are more likely to find comprehensible, and so a basis for selecting the sample density for a geostatistical survey, than alternatives such as kriging variance. Furthermore, it ~~is an intuitively meaningful~~appears to be a measure of uncertainty ~~, it~~which participants in the study found comprehensible, and so were able to use to select a grid sample spacing. It recognises that spatial variation means that maps interpolated from

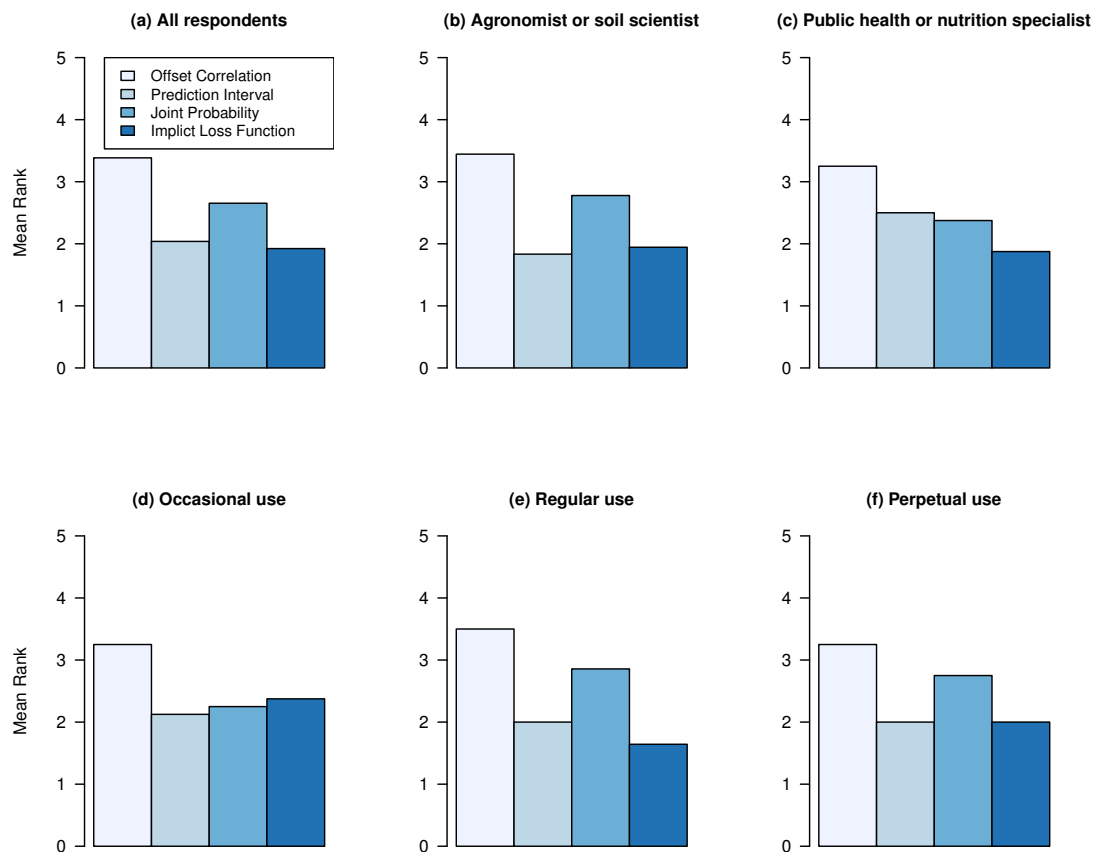



Figure 9. Ranking of test methods in terms on the most effective: (a) by all respondents, professional group: (b) agronomists or soil scientist and (c) public health or nutritionist specialists, and frequency of use of statistics: (d) occasional use, (e) regular use and (f) perpetual use.

575 offset grids will differ but that the more robust the sampling strategy the more consistent they will be. ~~There is a paradox here, however, in that the previous study Chagumaira et al. (2021) showed that~~ However, Chagumaira et al. (2021) found that measures of uncertainty related to a specific management threshold of the mapped variable were preferred by participants for the interpretation of ~~survey outputs in terms of uncertainty was easiest for stakeholders with measures related directly to a decision made with the information. The offset correlation is a general measure, and the absolute magnitude of uncertainty~~

580 ~~has greater bearing on a specific decision. Indeed, Lark and Lapworth (2013) proposed the offset correlation particulary with general baseline surveys in mind. There is more research needed to develop sound but accessible ways to engage stakeholders with uncertainty consistent across planning and~~ uncertain spatial information to general quality measures without a specific

management or policy implication. In this case, in contrast, the preferred criterion, the offset correlation, is a general measure of map quality, which is not directly linked to specific interpretation.

Joint probabilities were ranked second. Under this method, the information users selected spacings where joint probabilities was 1.0 or very close, i.e. the prediction equivalent to the overall mean. This suggests that the information users may not have fully understood the method. This finding is consistent with the general view that users of information commonly find probabilities difficult to interpret (Spiegelhalter et al., 2011). Because probabilities are bounded [0,1], the 'relation-to-reference' attribute effect by Hsee (1998) may explain the previous preference for joint probabilities (Jenkins et al., 2019; Chagumaira et al., 2021), but information users still struggle to interpret them correctly. Perhaps if the problem had been framed in a different way, the information users may have understood this method much better. More work is needed to investigate if framing the joint probabilities in a different way would improve the judgement of utility of the information users. More examples and more illustration may be needed in order to 'prime' the participants before the exercise.

Prediction intervals were ranked third by all the respondents, but there was no evidence against the null hypothesis of random selection among the available spacings. During a feedback session, the information users cited difficulties of assessing the significance of a given prediction interval given that it can be associated with different prediction values. For very large or small prediction values the uncertainty is immaterial, it is near decision threshold that it becomes important. Similarly, prediction intervals were not highly ranked by information users for communicating uncertainty in maps (Chagumaira et al., 2021). Similar reasons were given by the respondents. We expected that prediction intervals would be of greatest value for specific interpretation of particular sites but would be of limited value for survey planning.

The implicit loss functions was the lowest-ranked method. The group also commented that they had difficulties understanding this method, and most people opted for the central value. Loss functions are not readily accessible. It is difficult to define a loss function because it requires the cost of the errors, and we tried to show information users some consistent approach with some plausible design. The fact that they did not understand the loss functions, shows there is need for more specific examples to help information users think about loss function and their implications. It might help the information users to provide some quantitative information about the costs of the survey, cost associated with intervention campaigns and costs of the impacts on MNDs on a country's gross domestic production. A reflection of these would allow the information user to use these implicit assumptions when they were making decisions for selecting a fixed grid spacing for working with (Lark and Knights, 2015).

The background of the information users, i.e., professional group and frequency of use of statistics, had no influence on their responses for all the methods. However, the background of the information users had an influence on their ranking of the methods in terms of their effectiveness. The offset correlation was ranked as the most effective by all professional groups and by all respondents separated by frequency of use of statistics. Prediction intervals were ranked least effective by those respondents who identified as agronomist or soil scientist, but were ranked second by those in public health or nutrition.

At the ~~begin~~ beginning of the online workshop, we explained each method with the aid of illustrations. After an explanation of each method, there was a feedback session to allow the participants opportunities to seek clarity on ambiguous and unfamiliar

620   concepts from the presenters. The participants' questions were answered and explained in different ways by CC, RML and AEM, with the use of illustrations. However, there are limitations with online workshops. Most participants would have the cameras switched off, and the "unconscious" feedback to presenters by observing the reactions of participants could not be noticed as during in-person workshops. The "unconscious" feedback would prompt the presenter to use a different approach to explain unfamiliar concepts and ambiguous terms. Due to internet connectivity, online workshops are timed and there will less

625   time for feedback sessions. In such instances, respondents may seek clarity from the colleagues who have the same interests, resulting in bias (Ball, 2019).

All the methods may give different results for different ~~variable~~variables, because they depend on the variogram of the variable in question. There maybe different grid spacings selected for the different variables. A potential problem may exist, if the variables were to be sampled in one survey and what spacing should be used? This is an important question that needs

630   to be addressed when planning for soil and crop sampling. It may be reasonable to opt for the grid spacing for the variable that maybe the hardest to characterise. Another option would to consider some minimum quantile over all variables through a group elicitation. Black et al. (2008) proposed that a critical subset of soil properties are identified such that the overall sampling scheme is satisfactory for all of the so-called 'canary indicators'.

All the information users recruited in this study were employed in public sector institutes in roles (e.g., universities, civil

635   organisations, research, and extension) and had experience in their respective fields in an SSA setting. In terms of sample size, we have no prior basis to select a sample size because, as this was the first study of this topic, it was not clear how to select an appropriate effect size. As a result, our major consideration was recruiting individuals willing to participate and with experience in their respective institutions. We therefore attempted to recruit the entire set of suitable respondents in each country. In future work initial power analysis might be considered.

640   ## 5   Conclusions

In this study we evaluated four methods of communicating uncertainty associated with kriging predictions made from data from a geostatistical survey, to determine an appropriate sampling density to meet information users expectations. Users of information on soil variation need accessible ways of understanding the implications of sampling designs on spatial prediction and their uncertainties. The background (professional group and frequency of use of statistics) of the ~~stakeholder~~ information

645   user had no influence in the responses selected for each approach. Of these methods we tested, the offset correlation was most favoured, but had no direct link to decision making and some methods of communication were not well understood (~~conditional~~ joint probabilities and implicit loss functions). The offset correlation will likely be more useful to ~~stakeholders~~information users, with little or no statistical background, who are unable to express their requirements of information quality based on other measures of uncertainty. Although previous work has found that uncertainty of spatial information is best understood

650 when presented in terms of a decision-specific metric, that was not the case here. This shows that more work must be done to develop and elucidate decision specific approaches, perhaps through methods to elicit useful loss functions.

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Appendix A

Appendix A: Theory

655 In Appendix A of this paper, we describe in detail the test approaches.

A1   Prediction interval

Some unknown quantity at a location (e.g. soil pH or Se_{grain}) is characterised by a prediction distribution conditional on the data and statistical model. The kriging prediction is a weighted average of the data

$$\tilde{Z}(\mathbf{x}_0) = \sum_{i=1}^{N} \lambda z(\mathbf{x}_i), \tag{A1}$$

660 where $z(\mathbf{x}_i)$ is the data and λ are the kriging weights (Webster and Oliver, 2007). The kriging variance, σ_{K}^2 is defined as:

$$\sigma_{K}^2 = E[\{Z(\mathbf{x}_0) - \tilde{Z}(\mathbf{x}_0)\}^2]. \tag{A2}$$

Cross-validation predictions of the statistical model need to be examined by exploratory analysis of the kriging error, $\varepsilon(\mathbf{x}_0)$, defined as $\varepsilon(\mathbf{x}_0) = \{z(\mathbf{x}_0) - \tilde{Z}(\mathbf{x}_0)\}$ to check if the assumption of the normality holds. The kriging predictor is unbiased and the mean of the errors is zero, and their standard deviation is equal to the kriging standard deviation, σ_K, from kriging. Based 665 on this, a 95% prediction interval can be computed as:

$$\left[\tilde{Z}(\mathbf{x}_0) - 1.96\sigma_{K}(\mathbf{x}_0), \tilde{Z}(\mathbf{x}_0) + 1.96\sigma_{K}(\mathbf{x}_0)\right]. \tag{A3}$$

The prediction distribution may also be obtained on a block support–for example if predictions are required at the scale of a farm mean or a mean for an administrative region. The same approach holds to the derivation of a prediction interval.

A2   Joint probability

670 We can calculate the joint probability that a location requires an intervention, and that the kriged estimate does not indicate this. If $\mathbf{x}_0$ is the location of interest, $\tilde{Z}(\mathbf{x}_0)$ is the prediction and $z(\mathbf{x}_0)$ the value of the variable at $\mathbf{x}_0$, then $\tilde{Z}(\mathbf{x}_0) - z(\mathbf{x}_0) = \varepsilon(\mathbf{x}_0)$, the error of the kriging predictions. The covariance of $z(\mathbf{x}_0)$ and $\varepsilon(\mathbf{x}_0)$ is:

$$\text{Cov}\left[z(\mathbf{x}_0), \varepsilon(\mathbf{x}_0)\right] = \text{Var}\left[Z(\mathbf{x}_0)\right] - \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{c}, \tag{A4}$$

where $\boldsymbol{\lambda}$ denotes the vector of kriging weights for observations used to make the prediction, and $\mathbf{c}$ denotes the vector of covariances between each of these observations and $Z(\mathbf{x}_0)$. We can therefore, specify the joint distribution of $\{z(\mathbf{x}_0), \varepsilon(\mathbf{x}_0)\}$, assuming a normal random variable and prediction errors and conditional on the variance parameters of a geostatistical model. We also specify some $\mathbf{x}_0$ which will give a conservative output–e.g. for a square grid we could specify $\mathbf{x}_0$ at the centre of a grid cell where kriging variance is largest. From this it is possible to compute the joint probability that $\tilde{Z}(\mathbf{x}_0) \geq z_t$ given that $z(\mathbf{x}_0) < z_t$, i.e. the probability, given that an intervention is required at $\mathbf{x}_0$ that, due to error in prediction, the mapped variable does not show this. A detailed description of how the desired conditional probability can be obtained from the joint probabilities is presented in the Supplement.

A3 Implicit loss function

The loss is a function of the error, if $\tilde{Z}$ is the predicted value and the true value is z, then the error is $\mathcal{L}(\tilde{Z} - z)$. If the value of z is equals to 0, then the error is equal to $\tilde{Z}$. The loss function is explained in greater detail by Journel (1984), Goovaerts (1997) and Lark and Knights (2015). Journel (1984) defined a general linear loss function as:

$$\mathcal{L}(\tilde{Z} - z) = \alpha_1 |\tilde{Z} - z| \text{ if } \tilde{Z} < z$$
$$= \alpha_2 |\tilde{Z} - z| \text{ if } \tilde{Z} \geq z. \tag{A5}$$

The parameters α_1 and α_2 have positive real values. The coefficient α_2 is the loss per unit error of underestimation and α_1 is the loss per unit of error of overestimation. The slopes, α_1 and α_2 define the asymmetry of the loss function. The loss function can be symmetrical, i.e. penalizing overestimation and underestimation equally; or can be asymmetrical because over-and-underestimation have different consequences. The asymmetry of the loss function is the ratio of the loss per unit value by which a quantity is underestimated to the loss per unit value of an overestimation (Lark and Knights, 2015). The asymmetry, a, is obtained by

$$a = \frac{\alpha_2}{\alpha_1}, \tag{A6}$$

i.e., is independent of the absolute value of z. If the loss function depends only on the estimation error, then z can be set to zero, without loss of generality and the expected loss can be computed as a function of the error variance, and so of the sample size (Lark and Knights, 2015). Increasing sample size reduces the minimum expected loss in so far as it reduces the error variance. Therefore, the cost of obtaining n samples can be measured at which the marginal cost of an additional sample point is equal to the reduction in expected loss that single sample achieves (Goovaerts, 1997). However, it maybe difficult to define a loss function prior to making decisions about sampling. The losses may not be easy to quantify, e.g. social costs of failing to intervene, costs of unnecessary interventions, loss of confidence in the decision-making organisation. information

users can be helped to reflect on possible loss functions through the implicit loss function. It is a loss function that makes a specified sample size, n, a rational choice, given the marginal costs. That is to say, it is the loss function implied by a choice of $\bar{n}$, assuming this is rational. The implicit loss function is conditional on a logistic model, that expresses the marginal costs of the sampling exercise and the conditional distribution of z as a function of effort (Lark and Knights, 2015) and is obtained by finding $\bar{\alpha}_1$ (given asymmetry), such that

$$\check{\mathcal{L}}(\bar{n}-1|\bar{\alpha}_1, \bar{\alpha}_2, \boldsymbol{\phi}) - \check{\mathcal{L}}(\bar{n}|\bar{\alpha}_1, \bar{\alpha}_2, \boldsymbol{\phi}) = \text{C}(\bar{n}) - \text{C}(\bar{n}-1), \tag{A7}$$

where $\bar{n}$ is the specified number of samples, $\text{C}(n)$ is the function that returns the cost of n samples and $\boldsymbol{\phi}$ is a vector of variogram parameters, so kriging variance is a contributor. The asymmetry can be set at different values, or inferred from other elicited opinions of the information user group (Lark and Knights, 2015). The expected loss can be minimised at a location given some prediction distribution of $\tilde{Z}$ for the variable of interest by specifying the value of variable corresponding to a given probability (P_0), i.e.,

$$\tilde{Z} = F^{-1}(P_0). \tag{A8}$$

Where, F^{-1} denotes the quantile of the prediction distribution for a probability P_0 obtained from

$$P_0 = \frac{\alpha_2}{\alpha_1 + \alpha_2}, \tag{A9}$$

(Journel, 1984). Lark and Knights (2015) suggested that a information user group might consider an implicit loss function for different $\bar{n}$ as starting points in the elicitation of a sample size, or compare implicit loss functions for different projects given different partitions of a total budget between them. No attempt has been made to elicit opinions from information users on implicit loss function, so we tried it in this study.

A4   Offset correlation

The expected correlation between the kriging predictions, $\tilde{Z}_1(\mathbf{x}_0)$, made from data collected on a square grid, of interval ζ, and predictions, $\tilde{Z}_2(\mathbf{x}_0)$, made from a second grid, a translation of the first grid by $\zeta/2$ in both directions is known as the offset correlation. The correlation of the two kriging predictions can be computed by:

$$\rho_{\tilde{Z}_1, \tilde{Z}_2} = \frac{\mathbf{C}_{\tilde{Z}_1, \tilde{Z}_2}(\mathbf{x}_0)}{\sqrt{\sigma^2_{\text{K}_{\tilde{Z}_1}} \sigma^2_{\text{K}_{\tilde{Z}_2}}}}, \tag{A10}$$

where $\mathbf{C}_{\tilde{Z}_1, \tilde{Z}_2}(\mathbf{x}_0)$ is the covariance $\tilde{Z}_1(\mathbf{x}_0)$ and $\tilde{Z}_2(\mathbf{x}_0)$. $\sigma^2_{\text{K}_{\tilde{Z}_1}}$ and $\sigma^2_{\text{K}_{\tilde{Z}_2}}$ are the kriging variances of the predictions from the first and second grid, respectively.

The offset correlation depends on $\mathbf{x}_0$, and is smallest at the location furthest from points on either grid. This minimum offset correlation is used to evaluate predictions from a grid spacing ζ. As the uncertainty in the map, attributable to sample density, decreases, the offset correlation increases. The denser the grid the more consistent the maps and the offset correlation will be

1 if the maps are identical and 0 if they are entirely unrelated to each other. The offset correlation is bounded on the interval [0,1], and ranges from zero (when the maps produced from the two grids are independent of each other (at a coarse spacing) and approach 1 as the grid becomes finer and the two maps become increasingly similar. Lark and Lapworth (2013) describes the offset correlation in greater detail.

Appendix B: Contingency tables

In Appendix B we described how a contingency table can be partitioned to evaluate whether there are differences in the responses of the participants based on (i) variable used in the test method, (ii) professional group and (iii) by frequency of use of statistics. In Table B1, we illustrate how the contingency table can be partitioned. The table can be partition into components corresponding to pooled table and subtables of the full table.

The full table in Table B1, was partitioned into components corresponding to subtables for soil pH (Subtable 1 in Table B1), and Se_{grain} concentration (Subtable 2 in Table B1). Then the pooled table completes the partition. The degrees of freedom and deviances for the three table sum to the degrees of freedom and deviance of the full table. Using the contingency table, we could conclude if there are differences in responses for the two variables. The full table in can further be partitioned, in a similar way, by the background of the respondents i.e., professional group and frequency of use of statistics.

~~In this appendix, we present the~~ The full contingency table for Q1, for offset correlation, is presented as Table ~~A1~~B2, in the appendix. The table shows how many individuals selected the given responses for offset correlation. This table is according to variable used (soil pH vs. Se_{grain}), professional group and frequency of use of statistics. Table ~~A2~~ B3 shows how many individuals selected a given response to Q1, for offset correlation, when columns are pooled within variable used, soil pH or Se_{grain} concentration. Table ~~A3~~ B4 shows the pooled counts of the responses for Q1.

Table B1. An illustration of how the log-likelihood ratio was used to partition full table into subtables and pooled tables.

Deviance = L_f
Response = $O_{i,j}$
degrees of freedom = $DF_F = (2-1)\times(12-1) = 11$

Full table

Response	Soil pH						Se_{grain} concentration					
	Agronomy or soil science (AGS)			Public health or nutrition (PHN)			Agronomy or soil science (AGS)			Public health or nutrition (PHN)		
	Occasionally	Regularly	Perpetually	Occasionally	Regularly	Perpetually	Occasionally	Regularly	Perpetually	Occasionally	Regularly	Perpetually
Spacing 1	$O_{1,1}$	$O_{1,2}$	$O_{1,3}$	$O_{1,4}$	$O_{1,5}$	$O_{1,6}$	$O_{1,7}$	$O_{1,8}$	$O_{1,9}$	$O_{1,10}$	$O_{1,11}$	$O_{1,12}$
Spacing 2	$O_{2,1}$	$O_{2,2}$	$O_{2,3}$	$O_{2,4}$	$O_{2,5}$	$O_{2,6}$	$O_{2,7}$	$O_{2,8}$	$O_{2,9}$	$O_{2,10}$	$O_{2,11}$	$O_{2,12}$

Pooled table (Professional groups pooled within variable):
Deviance = L_{p1}
degrees of freedom = $DF_{P1} = (2-1)\times(2-1) = 1$

Pooled table 1

Response	Soil pH	Se_{grain} concentration
Spacing 1	$O_{1,1}+O_{1,2}+O_{1,3}+O_{1,7}+O_{1,8}+O_{1,9}$	$O_{1,4}+O_{1,5}+O_{1,6}+O_{1,10}+O_{1,11}+O_{1,12}$
Spacing 2	$O_{2,1}+O_{2,2}+O_{2,3}+O_{2,7}+O_{2,8}+O_{2,9}$	$O_{2,4}+O_{2,5}+O_{2,6}+O_{2,10}+O_{2,11}+O_{2,12}$

Subtable 1 (Soil pH):
Deviance= L_{s1}
degrees of freedom = $DF_{S1} = (2-1)\times(6-1) = 5$

Subtable 1

Response	Soil pH					
	Agronomy or soil science (AGS)			Public health or nutrition (PHN)		
	Occasionally	Regularly	Perpetually	Occasionally	Regularly	Perpetually
Spacing 1	$O_{1,1}$	$O_{1,2}$	$O_{1,3}$	$O_{1,4}$	$O_{1,5}$	$O_{1,6}$
Spacing 2	$O_{2,1}$	$O_{2,2}$	$O_{2,3}$	$O_{2,4}$	$O_{2,5}$	$O_{2,6}$

Subtable 2 (Se_{grain} concentration):
Deviance = L_{s1}
degrees of freedom= $DF_{S2} = (2-1)\times(6-1) = 5$

Subtable 2

Response	Se_{grain} concentration					
	Agronomy or soil science (AGS)			Public health or nutrition (PHN)		
	Occasionally	Regularly	Perpetually	Occasionally	Regularly	Perpetually
Spacing 1	$O_{1,7}$	$O_{1,8}$	$O_{1,9}$	$O_{1,10}$	$O_{1,11}$	$O_{1,12}$
Spacing 2	$O_{2,7}$	$O_{2,8}$	$O_{2,9}$	$O_{2,10}$	$O_{2,11}$	$O_{2,12}$

Deviance partition:
$L_f=L_{p1}+L_{s1}+L_{s2}$

Degrees of freedom partition:
$DF_f=DF_{p1}+DF_{s1}+DF_{s2}$

Table B2. The full contingency table for Q1 for offset correlation, showing how many respondents selected the given responses for offset correlation. The table is according to variable used, professional group and frequency of use of statistics in job role. The figures in parentheses are the expected numbers, $e_{i,j}$ a product of row and column totals dived by the total number of responses. The acronyms represent the professional groups (AGS–agronomist or soil scientist; PHN– public health or nutrition specialists), and frequency of use of statistics in job role (Pep– perpetual use of statistics; Occ–Occasional use of statistics and Reg– regular use of statistics).

Response	soil pH						Se$_{\text{grain}}$					
	AGS			PHN			AGS			PHN		
	Pep	Occ	Reg	Pep	Occ	Reg	Pep	Occ	Reg	Pep	Occ	Reg
Offset=0.4	0(0.23)	0(0.31)	1(0.85)	0(0.08)	0(0.31)	0(0.23)	0(0.23)	2(0.31)	1(0.31)	0(0.08)	0(0.31)	0(0.23)
Offset=0.5	0(0.17)	1(0.23)	0(0.63)	0(0.06)	0(0.23)	1(0.17)	0(0.17)	0(0.23)	1(0.23)	0(0.06)	0(0.23)	0(0.17)
Offset=0.6	1(0.40)	0(0.54)	1(1.48)	0(0.13)	1(0.54)	0(0.40)	1(0.40)	0(0.54)	0(0.54)	0(0.13)	1(0.54)	2(0.40)
Offset=0.7	1(0.92)	2(1.23)	2(3.38)	0(0.31)	3(1.23)	2(0.92)	1(0.92)	1(1.23)	3(1.23)	0(0.31)	1(1.23)	0(0.92)
Offset=0.8	0(0.87)	0(1.15)	5(3.17)	1(0.29)	0(1.15)	0(0.87)	0(0.87)	1(1.15)	5(1.15)	1(0.29)	1(1.15)	1(0.87)
Offset=0.9	1(0.40)	1(0.54)	2(1.48)	0(0.13)	0(0.54)	0(0.40)	1(0.40)	0(0.54)	1(0.54)	0(0.13)	1(0.54)	0(0.40)

Table B3. A subtable showing how many individuals selected a given response to Q1, for offset correlation, when columns are pooled within variable used (soil pH vs. Se_{grain} concentration).

Response	soil pH	Se_{grain}
Offset=0.4	1	3
Offset=0.5	2	1
Offset=0.6	3	4
Offset=0.7	10	6
Offset=0.8	6	9
Offset=0.9	4	3

Table B4. Pooled responses given to the question on offset correlation.

Response	Pooled counts
Offset=0.4	4
Offset=0.5	3
Offset=0.6	7
Offset=0.7	16
Offset=0.8	15
Offset=0.9	7

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Competing interests. The authors declare that they have no conflict of interest.

Disclaimer. The funders were not involved in the design of this study or the collection, management, analysis and interpretation of the data, the writing of the report or the decision to submit the report for publication.

Ethics statement. Ethical approval to conduct this study was granted by the University of Nottingham, School of Biosciences Research Ethics Committees (SBREC202122022FEO) and participants gave informed consent to their participation and subsequent use of their responses.

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