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Interactive comment on "GAIA 5.0 – A five-dimensional geometry for the 3D visualization of Earth' climate complexity" *by* Renate C.-Z.-Quehenberger et al.

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Dear reviewers,

many thanks for your time and consideration also on behalf of all co-authors. We are grateful for your valuable inputs and suggestions to improve our publication. Please allow me a final remark concerning the problem our co-author Sergio Rubin brought up and which might also be of common concern since we use the term "Poincaré homology sphere" in our Gaia 5.0 video and publication:

Indeed, our geometrical concept is not linked to Perelman's work which commonly ap-

preciated as proof of the Poincaré conjecture. We are strictly relying on Poincaré's idea's, his search for the fundamental polyhedron which we assume to having identified as 3D representation of the Penrose Kites & darts tiling (E±), the unit cell of the 5-dimensional space,– the space which Poincaré himself found appropriate for group theory (Poincaré, 1899, p.19). If we look at attached images of the visualisation of the epita-dodecahedron, – a simply connected (?) manifold of golden triangles forming the shape of a 4-dimensional dodecahedron where the blue circle decoration (fig.1) is forming elliptic curves in motion (fig.2). It reminds us very much on Poincaré's sentence: "The analogy with the formation of cycles in the theory of Fuchsian groups is evident. It is even stronger if we assume there is only a single polyhedron P1 ." – This is also how we may imagine the one-dimensional linear flow attached to the polyhedral space, where the finite group of rotations SO(4) acts freely in the tangent (beyond the red sphere, while the inner space belongs to S2) on S3. The picture of the dodecahedral space with circles in motion is clearly matching our picture of the hypersphere which we use as a model for the Earth is built from icosahedral symmetries,– the dual of the dodecahedron (fig.3). Distinct to Poincaré's approach Perelman's work is based on Hamilton's Ricci flow method that acts on a 3-manifold with a Riemannian metric. For this he introduced celebrated new algebraic tools in differential geometry that found many applications on dynamical systems since then. While Perelman's work did give convincing arguments for eliminating Hamilton's problem with the cigar soliton, he did not claim to have proven the Thurston Geometrization Conjecture – which was an attempt to prove the Poincaré conjecture – until the end of a second even more difficult paper, "Ricci Flow with Surgery on Three Manifolds" (2003) but he never claimed for what he is credited nor did he ever refer to Poincaré's original problem in any of his three seminal papers (Perelman, 2002, 2003a b). As John Stillwell (2009) emphasized, Poincaré's analysis situs treatises are very different from what we know as Perelman's solution to the Poincaré conjecture that involves physical properties such as time, entropy and heat.

Although there is a common ground between Ricci and Poincaré their works are very different: Poincaré is clearly building on his experience with Fuchsian groups in the early 1880s (see Poincaré (1985) where each group is associated with closed paths in a 3-manifold defined by a polyhedral region with faces identified by certain geometric transformations (Poincaré, 1892). Also Gregorio Ricci was influenced by Fuchs when he wrote a thesis on differential equations, entitled "On Fuches's Research Concerning Linear Differential Equations" With Tullio Levi-Civita he worked on the "absolute differential calculus" with coordinates or tensor calculus on Riemannian manifolds. But both work on very different approaches: Ricci presented a system of point transformations in order to determine covariant functions in differential geometry while Poincaré's mathematical edifice is clearly built on higher dimensional spacial formations, dedicated to group theory permutations of polygons and polyhedra and the search for the fundamental polyhedron.

Our construction of the hypersphere is very much conform with the horosphere, a Euclidean sphere tangent to the unit sphere where the point X of tangency is the center of the horosphere. Also his "ideal, hyperbolic simplex" with dihedral angles 60° of which he takes two copies and glues the faces together is conform to one side of our polyhedra E$\pm$ face we suggest. Furthermore E$\pm$ is exhibiting the Poincaré wished.

Alsing, Miller, T.-S.Yau et al, described the Ricci Flow of a piecewise-flat simplicial geometry. Their model introduces piecewise-flat 3-geometries considered here are built of isosceles-triangle-based frustum blocks (solids that lie between one or two parallel planes cutting it) with regular icosahedra as S2 cross-sectional geometries (fig.4). They remind us of our irregular pyramids E$\pm$ which are whose edges are composed of parallels of the grid Z5 and can be composed to icosahedra (as shown in the video Gaia 5.0 as model for the formation of aerosol).

Quite a few more arguments (for which here is no space) would be available in support of our claim as well as even more open questions concerning our visualisation (after the description of Threlfall and Seifert, 1933 with twists of opposite lying faces but not by $3\pi/5$ radians but by $\pi/5$ radians as in the case of spherical dodecahedron space).

We hope we will able to discuss it one day with the mathematical community in order to justify our use of the term "Poincaré homology sphere" in the here discussed paper as model for the Earth complex dynamic system and elsewhere.

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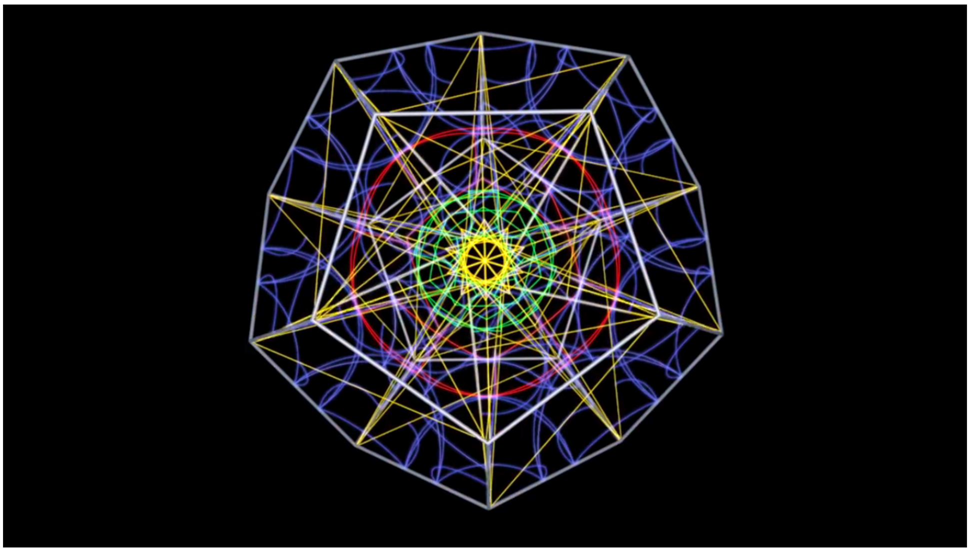

Fig. 1. Epita-dodecahedron with blue horocycles in motion flow (RCZQ, QC, 2012)

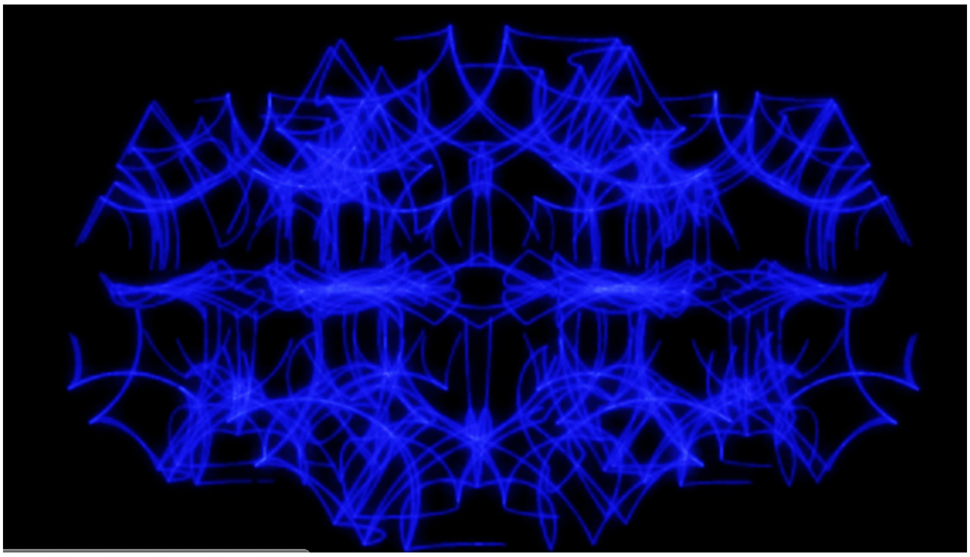

Fig. 2. Horocycles of the Epita-dodecahedron in flow (RCZQ, QC, 2012)

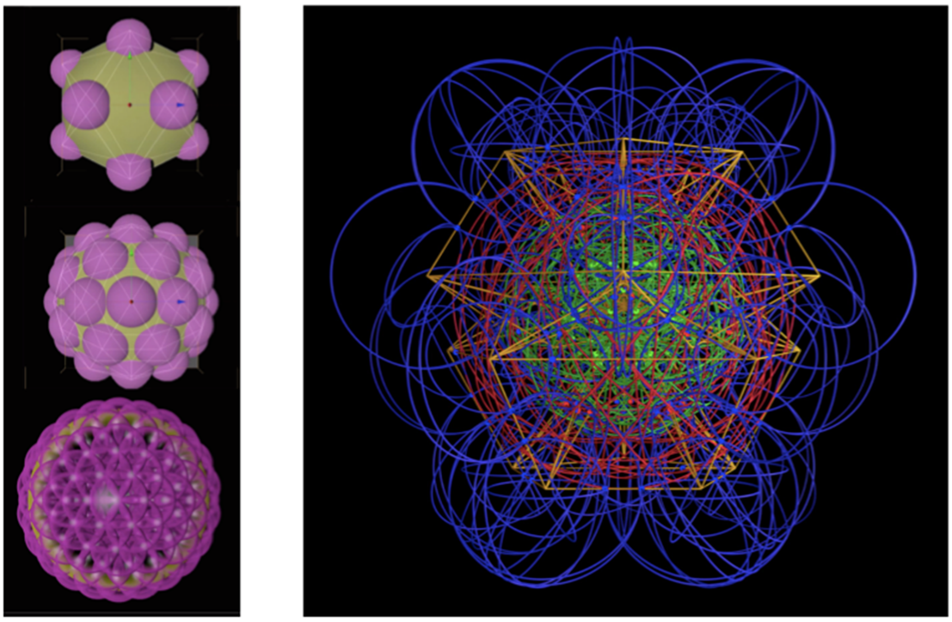

Fig. 3. Hypersphere construction with icosahedral symmetries + Homology sphere (RCZQ, QC,2012)

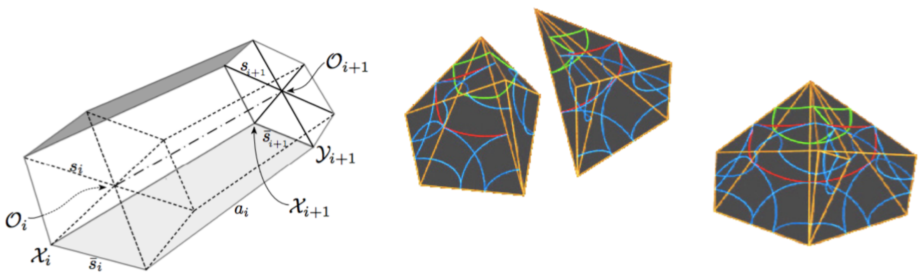

Fig. 4. a) The six identical isosceles frustum blocks (Alsing, Miller, Yau et al, Embedding Ricci flow) b) Two E± sharing together a hexagonal basis, RCZQ, 2013)