Supplement: Preliminaries of 5D geometry

Some paragraphs with a detailed description about the philosophical and geometrical approach of a 4-dimensional spacio-temporal continuum embedded into a 6D-framework.

**1. Spirals and Fractals in the geometry of the 5-dimensional space**

Spirals of galaxies and cyclones are exhibiting self-similar fractal features like the branching of rivers, veins and neurons. Similarly we find in the turbulence of air in the vortices of taiphoons in smaller regions of air. The empirical logarithmic approximation is shown with several examples of satellite and radar images of a tropical cyclone (TC) (Yurchak, 2007). The manifold role of the golden ratio 1:1/2(1± √5) (= Phi, 1,681… or phi,0, 618…) and related logarithmic spirals have scientific significance in science (f.e. in astronomy, biology, genetics, etc.) as well as topics of many popular „golden ratio“ books. The ratio of two consecutive Fibonacci numbers (1,1, 3, 5, 8, 13, 21...) approaches the Golden Ratio. Fibonacci series have been recently (again) raised as a principle of biological organization (Petoukhov 2016; Petoukov and He, 2010; Rapoport & Pérez, 2018).
The pattern of the golden ratio, in its 2-dimensional form as known as Penrose Tilings (Penrose, 1974) also known to represent the 2D slice of the 5-dimensional space and serves as models of quasicrystals. The 3D representation of the Penrose Kites & Darts tiling allows to visualize the infinite 5-dimensional space with the boundary of a dodecahedron. The self-similarity at every scale reminds us fractals and that ultimate chaos results in the pattern of the Golden ratio.

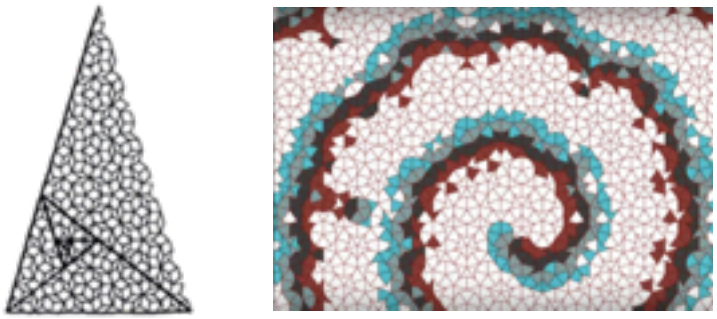


Figure 1 a) Generation principle of the Penrose Pattern depicted by means of a golden triangle b) Logarithmic distribution of triangles (Source: Jay Kappraff, Connections Fig. 20.6, p. 541) c) 4-fold rotation of the 4D -Figure 2 a) representation of the Penrose kites & darts pattern, named epitahedron E+ (Quehenberger, 2014)

Hence both characteristics of cyclones, their fractal and chiral nature could be explained by the assumption of an underlying dynamic higher-dimensional space. The concept of a discrete space as visualized by the epita-dodecahedron comprises the geometrical fact that he faces of the space cells can be enfolded into smaller and smaller regions resulting in a fractal nature of space. The logarithmic spiral is a generation method for Penrose Patterns. (Kappraff, 2001)
Chirality remains also an intrinsic feature of the fractal higher-dimensional dodecahedral space created by the 3D representation of the Penrose Kites & Darts Tiling. Therefor any chiral and fractal characteristics of shapes and dynamics could be traced back to this feature of space.
However, the idea to attribute the geometrical features of 5-dimensional space in this way was not done before. Although we see similar approaches by Descartes and Kepler it was never recognized as such. The 5D theories Kaluza

(1921) and Klein who conceptualized it as a small circle with diameter $10^{-30}$cm and as enrolled cylindric space (1926) and de Broglie, and Pauli, several attempts by Einstein et al. are widely forgotten after a short revival in string theories.

**2 Space and time -- the lost dimension**

The French philosopher Paul Virilio (1932--2018) dedicated a book to the »Lost Dimension« in which he diagnoses the mental perception of space in the 20th century as negation of the notion of physical dimension (Virilio,1984). In the last decades also the notions of space and time as fundamental principles were put under question.

Before we introduce here the concept of an ontologic higher-dimensional space which might sound strange or even awkward to some readers, we ave to mention the current crisis in physics and the discussion about wether space and time (or just time) are emergent (cf. Lee Smolin, Christoph Wüthrich, Carlo Rovelli et al.).

It roots in the general covariance (point coincidence) in relativity theory, when Einstein noted, "Thereby time and space lose the last remnant of physical reality." (Einstein, 1916).

This lack of of a „real 4th dimension" seemingly results in human's cognitive lack of their ontologic position (Sloterdijk). The epistemic framework of the space where we are living is missing. This may be related to the notion „critical zone" for the biosphere. Therefor we consider it as extremely important to create a cognitive framework, that provided a space continuum.

This diagnosis of a century long erroneous development will not be corrected with one stroke. Nevertheless I hope it will flow into current research discussions.

As we all know Relativity theory lead to the merging of space and time and the introduction of relations between events in space-time. It is said that,

> 'in Newtonian physics, if we take away the dynamical entities, what remains is space and time. In relativistic physics, if we take away the dynamical entities, nothing remains. We have learned so far to give up the notions of ''space and time entities'' entirely since the "world is made by dynamical fields.'(Rovelli, 2006).

Recall that the notion of fields goes back to Michael Faraday (1791—1867) who made drawings and artistic experiments and developed with Clark C. Maxwell the electro-magnetic theory. It is based on the assumption of a space filling medium, or 'lines of force' which were his fabric of space.

> "Since at every point of space such a direction may be found, if we commence at any point and draw a line so that, […] the resultant force at that point, this curve will indicate the direction of that force for every point through which it passes, and might be called on that account a line of force. We might in the same way draw other lines of force, till we had filled all space with curves indicating by their direction that of the force at any assigned point." (Maxwell, 1855)

In the 5-dimensional space model we can use the parallels of the cubic grid lines for representing those 'lines of force' of which Faraday and Maxwell were pondering if they should assume them as real or not.

We may assign straight parallel lines into 5 directions of space ( c-.f. Ammann bars, the one-dimensional version of the Penrose Pattern) to electricity and curved lines to magnetism. Hence we get a space — similar to the phase space used in meteorology — where each point in the 4-dimensional space around the 3D sphere can be assigned to an electric scalar potential and a magnetic vector potential into a single four-vector.

*Spacetime* and the problem of time was discussed since the 1960s in the context of 3-geometries used in the program of reformulation and unification called 'geometrodynamics' with the request for "first principles"(Wheeler,1967). The here presented embedding of the Hamiltonian as "first principles" into the framework of 5-dimensional space meets attempts like assigning groups of motions to a rotating 4-hyperboloid which are discussed in quantum gravity now (c.f.Hojman-Kuchar-Teitelboim, 1974). From an artist-perspective the epistemic problem can be identified as 1) a mental problem

tied to dogmatic restrictions and 2) deficits in visual comprehension, since 3-geometries remained an abstract formalism and non-Euclidean and projective geometries with pseudospheres and hyperbolic planes cannot give the full picture of a spacial continuum. Currently popular books emphasize the S3 unit sphere as model for the Earth, there is no picture for it (c.f. Unzicker, 2019).

From a phenomenological point of view we my state that we have to consider the 3D space on the sphere as belonging to the next higher order of space which is the 4th dimension. If the planet would have merely the features a 3-dimensional ball, we would be forced to believe in antipodes: Then only people on the upper hemisphere would stand upright, Australians hanging on the sphere with heads downwards etc. Only a 4-dimensional sphere creates the illusion that *every point on the sphere is on top*. (Explanation of a 6 year old child, R.Q.).

## 1.1 Some geometrical preliminaries for the depiction of time in the 4th dimension

However, we can also think of the Earth as hypersphere analogously to a four-dimensional hypercube. A hypercube is a compound of 3D cubes on each face of a cube in the center moving in six degrees of freedom; the associated matrices correspond to the special orthogonal group of order 4, SO(4).. In the hypersphere at every point of the surface of the inner sphere a hemisphere is located. Then we my regard the space of the hemisphere in imaginary movement with the radius of the horizon as comparable with the upper half plane of the Poincaré space in the projective geometry. All those spaces in imaginary movement, depicted as counter-movement of the two small spheres in fig. 15.b (which depict the Lorentz transformation upon a Hamiltonian description) together are forming the 4-dimensional 'tangent space' of the hypersphere S3.

From a phenomenological point of view we have to experience the 3D space, -- the biosphere which we are inhabiting together with animals and plants on Earth and in air -- as the hyperbolic space over the imaginary boundary of the horizon. Alle these spaces together on the sphere as belonging to the next higher order of space which is the 4th dimension. The loss of the horizon is a frequent topic in contemporary arts, but was already a theme of the middle ages.

## 2,1 A phenomenological clarification of time and the 4th dimension

The French mathematician Joseph-Louis Lagrange (1736-–1813) first mentioned the 4th dimension as principle of mechanics long ago:

"Hence, one may regard mechanics itself as a geometry of the 4th dimension." (Lagrange, 1797, p. 223)

This fact is not so commonly known, because the Lagrangian function uses generalized coordinates for generating the correct equations of motion which summarizes the dynamics of the entire system and characterizes the possible motion of the particles also applied to meteorology.

The 19th century was dedicated to exploring the 4th dimension. A phenomenological explanation of the 4th dimension was first given in 1846 by the German philosopher, physicist and experimental psychologist Gustav T. Fechner's (1801–1887) in his ironical, funny philosophical essay *The 4th dimension* (Fechner, 1846). There he describes the imaginary movement of a row of houses ( fig.2. b) as countermovement to the rotating Earth which brings us every instant in a new space while we consider ourselves as being in rest. That's why we need the 4th dimension !

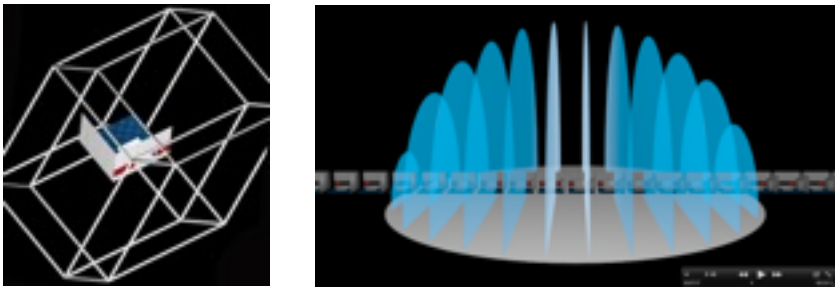


Figure 2 a) 4D Hypercube with house b) illustration of the 4th dimension, a row of houses changing space over time after G.T. Fechner, (still frames from "Passage to Level-1" , R.Q, QC: vimeo.com/39339348 )

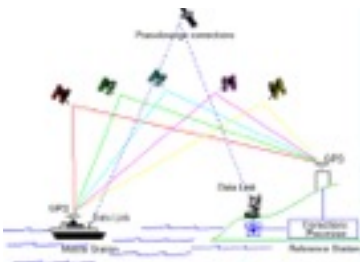
Fechner's "shadow-beeings" inspired Edwin Abbott Abbott (1838--1926) to his influential mathematical satire «Flatland -- A Romance of Many Dimensions« (1884 ) in order to illustrate that mathematicians are hardly able to imagine a sphere, not to speak of any higher dimensional object. »Flatland« is still referred to as "proof" of a "natural in-imaginability of higher dimensions" in mathematics books, -- despite his plea,

> "Yet I exist in the hope that these memoirs ... may find their way to the minds of humanity in Some Dimension, and may stir up a race of rebels who shall refuse to be confined to limited Dimensionality." (Abbott, 1884)

Around the same time, amidst popular discussions about the 4th dimension, Alicia Boole-Stott (1860-1940), daughter of the mathematicians Mary Everest and George Boole, the *The Princess of Polytopia*, who later conveyed her knowledge to the young H.S.M.Conway, first visualized all 4-dimensional Platonic solids (Boole-Stott, 1900). Her friend H. G. Wells should then *invent* a »Time Machine«, in order to travel on the time-axis towards a distant future (Wells, 1895). Hence, the debate about time travel is still a serious issue in physics while William Rowan Hamilton' (1805--1865) mathematical foundations for it in the »Algebra as the Science of Pure Time« (Hamilton, 1837) seem to be wildly forgotten. Together with the writer and poet Edgar Allen Poe (1809--1849) who proclaimed the expansion of the universe science-critical cosmogony »Eureka « (1848) -- which Einstein was reading in 1933 and 1940 before he agreed to Lemaître's idea of an expanding Universe -- these three artists probably produced the most eminent influence on science of the last 120 years (van Slooten, 2017). -- What about vice versa?

In his »Lectures on Quaternions« (1853) Hamilton noted that complex numbers (quaternions) are a composite of real and imaginary numbers ($z = x + iy \in C$ where $i = \sqrt{-1}$) living in four-dimensional space, -- a piece of the *curved space* surrounding a sphere. Therefore time and the 4th dimension is intrinsically connected to the surface of a sphere,--

Although distinct from the relativistic curved--space-time-concept following from Minkowski's "pseudo-4-dimensional" geometry in which realized Einstein dream of traveling on a beam of light, the principle of reality can be expressed in a quaternionic way (Silberstein, 1912).



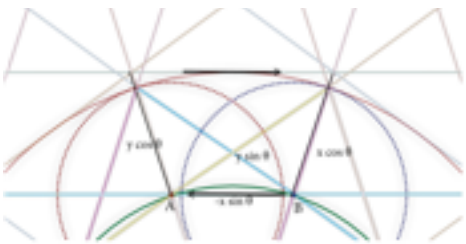


Figure 3. a) Image: Real time DGPS (GPS Tutor, 1998), https://nptel.ac.in) b) The Lorentz transformation embedded in the 4D space; A and B in the grid of the Ammann bars, the discrete space elements for a dynamic representation of the factor √-1, z = iy, (sketch RQ 2013)

By relying on Hamilton, the most important characteristics of a 4-dimensional system is the imaginary factor √-1 which is sometimes hidden or left out. F.e. Stokes shows how to omit √-1 in his equations on frictions of fluids (Stokes,1850, 15). Lord Kelvin needs to put 'properly the imaginary constituents' on the equations of the *Kelvin waves* (Thomson, 1879). If we compare the GPS set-up (3.a) with the quaternionic description (3.b) it is easy to see the geometrical difference of the technological realization of Minkowski's principle to the Hamilton's quaternionic principle of time. This conforms also to Robert Rosen's call for the requirement of a new concept:

> 'The surface of the sphere is in some sense a limit of its planar approximations, but to specify it in this way **requires a new concept (the topology of the sphere)** that cannot be inferred from local planar maps alone.' (Rosen, 1985)

We suggest the topology of the hypersphere relying on Silberstein's quaternionic formulation of relativity. As Conway notes in his letter to the editors attached to the Silberstein paper,

> „Beyond this, however, the quaternion has the advantage of being asymmetrical, the time-scalar occupying a different position from the space-vector. It is thus more in touch with real phenomena." (Conway's comment to Silberstein,1912)

However, we can also think of the Earth as hypersphere analogously to the four-dimensional hypercube. A hypercube is a compound of 3D cubes on each face of a cube in the center moving with degrees of freedom. In the hypersphere at every point of the surface of the inner sphere a hemisphere is located. Then we my regard the space of the hemisphere in imaginary movement with the radius of the horizon as comparable with the upper half plane of the Poincaré space in the projective geometry. All those spaces in imaginary movement, depicted as counter-movement of the two small spheres in fig. 3.b (which depict the Lorentz transformation upon a Hamiltonian description) together are forming the 4-dimensional 'tangent space' of the hypersphere $S^3$.

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